SDP as a tool for quantum information

Lecture 2: Dual certificates, strong duality, and numerical SDPs

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Lecture 2: Dual certificates and computation

6. Duality theory 2.0

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7. Analytical applications

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8. Numerical solvers

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9. Computer-assisted proofs

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9. Computer-assisted proofs
10. Outlook

Recap

Primal points achieve; dual points certify

PRIMAL – achievable

$$\begin{aligned} \alpha := \sup_{\rho} \quad & \mathrm{tr}(A\rho) \\ \text{subject to} \quad & \Lambda(\rho) = B \\ & \rho \succeq 0 \end{aligned}$$

DUAL – upper bound

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Feasible points are attainable strategies

DUAL – upper bound

$$\begin{aligned} \beta := \inf_{Y} \quad & \operatorname{tr}(BY) \\ \text{subject to} \quad & \Lambda^\dagger(Y) \succeq A \\ & Y = Y^\dagger \end{aligned}$$

Feasible points are rigorous upper bounds

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Feasible points are rigorous upper bounds

$\alpha \leq \beta$ (weak duality)

State discrimination is our running SDP

An ensemble $\{p_i, \rho_i\}_{i=1}^N$ is given; the measurement is the variable

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PRIMAL – achievable

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\text{subject to} \quad & \sum_{i=1}^N M_i = \mathbb{1} \\
& M_i \succeq 0 \qquad \text{for every } i
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Linear objective + affine normalisation + PSD constraints $\implies$ SDP

6. Duality theory 2.0

From upper bounds to strong duality

The dual problem for state discrimination

PRIMAL – achievable

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p^\star_{\text{succ}} := \max_{\{M_i\}} \quad & \sum_{i=1}^{N} p_i \operatorname{tr}(M_i \rho_i) \\
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& \sum_{i=1}^{N} M_i = \mathbb{1} & \longleftrightarrow \quad & Y = Y^\dagger
\end{aligned}
$$

Write the Lagrangian and factorise M_i

PRIMAL – achievable

$$p^\star_{\text{succ}} := \max_{\{M_i\}} \quad \sum_i p_i \operatorname{tr}(M_i \rho_i)$$

$$\text{subject to} \quad M_i \succeq 0, \quad \sum_i M_i = \mathbb{1}$$

$$L(\{M_i\}, Y, \{\sigma_i\}_i) := \sum_i p_i \operatorname{tr}(M_i \rho_i) + \operatorname{tr}\left[Y\left(\mathbb{1} - \sum_i M_i\right)\right] + \sum_i \operatorname{tr}(\sigma_i M_i)$$

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$$= \operatorname{tr}(Y) + \sum_i \operatorname{tr}[M_i\,(p_i\rho_i - Y + \sigma_i)]$$

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DUAL – upper bound

minimise $\quad \operatorname{tr}(Y)$

subject to $\quad p_i\rho_i - Y + \sigma_i = 0 \quad \forall i$

$\quad \sigma_i \succeq 0 \quad \forall i$

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Eliminate slack variables

$$p_i \rho_i - Y + \sigma_i = 0 \quad \iff \quad \sigma_i = Y - p_i \rho_i \qquad \forall i$$

Eliminate slack variables

$$p_i\rho_i - Y + \sigma_i = 0 \quad \Longleftrightarrow \quad \sigma_i = Y - p_i\rho_i \qquad \forall i$$

Since $\sigma_i \succeq 0$,

$$\sigma_i = Y - p_i\rho_i \succeq 0 \quad \Longleftrightarrow \quad p_i\rho_i \preceq Y \qquad \forall i$$

σ_i is a redundant slack variable

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DUAL – upper bound

$$\beta := \min_{Y} \quad \operatorname{tr}(Y)$$

subject to $\quad p_i\rho_i \preceq Y \quad \forall i$

State discrimination problem from Lecture 1

$$p_1 = p_2 = \frac{1}{2}, \qquad \rho_1 = |0\rangle\langle 0|\,, \qquad \rho_2 = |+\rangle\langle +|$$

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$$|m_1\rangle = \cos\left(\frac{\pi}{8}\right)|0\rangle - \sin\left(\frac{\pi}{8}\right)|1\rangle, \qquad |m_2\rangle = \sin\left(\frac{\pi}{8}\right)|0\rangle + \cos\left(\frac{\pi}{8}\right)|1\rangle, \qquad M_i = |m_i\rangle\langle m_i|$$

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$$\begin{aligned} p_{\text{succ}}(\{M_1, M_2\}) &= \frac{1}{2}\operatorname{tr}(\rho_1 M_1) + \frac{1}{2}\operatorname{tr}(\rho_2 M_2) \\ &= \cos^2\left(\frac{\pi}{8}\right) \\ &= \frac{2+\sqrt{2}}{4} \end{aligned}$$

This proves achievability; optimality still needs an upper bound

A dual operator reaches the same value

DUAL – upper bound

$$\begin{aligned} \min_{Y} \quad & \operatorname{tr}(Y) \\ \text{subject to} \quad & \frac{1}{2}|0\rangle\langle 0| \preceq Y \\ & \frac{1}{2}|+\rangle\langle +| \preceq Y \end{aligned}$$

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$$\min_{Y} \quad \mathrm{tr}(Y)$$

subject to

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$$Y^\star := \frac{1}{4}\left(\rho_1 + \rho_2 + \frac{\mathbb{1}}{\sqrt{2}}\right)$$

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$$\mathrm{tr}(Y^\star) = \frac{1}{4}\left(1 + 1 + \frac{2}{\sqrt{2}}\right) = \frac{2+\sqrt{2}}{4}$$

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$$Y^\star - \frac{1}{2}\rho_1 = \frac{1}{2\sqrt{2}} M_2 \succeq 0,$$

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$$Y^\star - \frac{1}{2}\rho_1 = \frac{1}{2\sqrt{2}}M_2 \succeq 0, \qquad Y^\star - \frac{1}{2}\rho_2 = \frac{1}{2\sqrt{2}}M_1 \succeq 0$$

$Y^\star$ is dual feasible, hence $\mathrm{tr}(Y^\star)$ is an upper bound.

Matching bounds prove optimality

$$\frac{2+\sqrt{2}}{4} = p_{\mathrm{succ}}(\{M_1, M_2\}) \leq p^{\star}_{\mathrm{succ}} \leq \mathrm{tr}(Y^{\star}) = \frac{2+\sqrt{2}}{4}$$

Matching bounds prove optimality

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Strong duality

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But when does strong duality hold?

Slater's theorem

Slater conditions

If the primal is bounded above and strictly feasible, that is,

$$\alpha < +\infty \qquad \text{and} \qquad \exists\, \rho \succ 0 \text{ such that } \Lambda(\rho) = B,$$

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Two-sided Slater conditions

If the primal and dual are strictly feasible, then $\boxed{\alpha = \beta}$.

State discrimination satisfies Slater conditions

PRIMAL – achievable

$$M_i = \frac{\mathbb{1}}{N} \succ 0, \qquad \sum_{i=1}^{N} M_i = \mathbb{1}$$

DUAL – upper bound

State discrimination satisfies Slater conditions

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$$M_i = \frac{\mathbb{1}}{N} \succ 0, \qquad \sum_{i=1}^{N} M_i = \mathbb{1}$$

DUAL – upper bound

$$Y = \lambda \mathbb{1}, \qquad \lambda > \max_i p_i$$

$$Y - p_i \rho_i \succ 0 \qquad \forall i$$

State discrimination satisfies Slater conditions

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$$M_i = \frac{\mathbb{1}}{N} \succ 0, \qquad \sum_{i=1}^{N} M_i = \mathbb{1}$$

DUAL – upper bound

$$Y = \lambda \mathbb{1}, \qquad \lambda > \max_i p_i$$

$$Y - p_i \rho_i \succ 0 \qquad \forall i$$

Strong duality holds, and both optima are attained

$$p^\star_{\text{succ}} := \max_{\substack{M_i \succeq 0 \ \forall i \\ \sum_i M_i = \mathbb{1}}} \sum_i p_i \operatorname{tr}(\rho_i M_i) = \min_{Y \succeq p_i \rho_i \ \forall i} \operatorname{tr}(Y)$$

7. Analytical applications

A useful certificate from the identity

A universal dimension bound

Theorem

For any ensemble $\{p_i, \rho_i\}_{i=1}^{N}$ on a d-dimensional Hilbert space, $p^{\star}_{\mathrm{succ}} \leq d \max_i p_i$.

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Hence,

$$p_i \rho_i \preceq p_i \mathbb{1} \preceq \left(\max_i p_i\right) \mathbb{1} = Y$$

A universal dimension bound

Theorem

For any ensemble $\{p_i, \rho_i\}_{i=1}^N$ on a d-dimensional Hilbert space, $p_{\mathrm{succ}}^\star \leq d \max_i p_i$.

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Hence,

$$p_i \rho_i \preceq p_i \mathbb{1} \preceq \left(\max_i p_i\right) \mathbb{1} = Y$$

Thus $Y = \left(\max_i p_i\right) \mathbb{1}$ is dual feasible and $\mathrm{tr}(Y) = d \max_i p_i$ is an upper bound. □

Uniform priors reduce the bound to d/N

Suppose

$$p_i = \frac{1}{N} \qquad i = 1, \ldots, N$$

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$$Y = \frac{\mathbb{1}}{N} \succeq \frac{\rho_i}{N} \qquad \text{for every } i$$

For uniform priors, $p^\star_{\text{succ}} \leq \operatorname{tr}(Y) = \dfrac{d}{N}$.

The most discriminable states

Theorem

Let $\{|\psi_i\rangle\}_{i=1}^N$ be pure states in $\mathbb{C}^d$. If

$$\frac{1}{N}\sum_{i=1}^{N} |\psi_i\rangle\langle\psi_i| = \frac{\mathbb{1}}{d},$$

then, for uniform priors, $p^\star_{\mathrm{succ}} = \dfrac{d}{N}$.

The most discriminable states

Proof. Set $M_i := \dfrac{d}{N} |\psi_i\rangle\langle\psi_i|$. Then $M_i \succeq 0$ and $\sum_i M_i = \frac{d}{N} \sum_i |\psi_i\rangle\langle\psi_i| = \mathbb{1}$, so $\{M_i\}$ is a POVM.

$$
\begin{aligned}
p_{\text{succ}}(\{M_i\}) &= \frac{d}{N^2} \sum_{i=1}^{N} \operatorname{tr}(|\psi_i\rangle\langle\psi_i|\,|\psi_i\rangle\langle\psi_i|) \\
&= \frac{d}{N}.
\end{aligned}
$$

The most discriminable states

Proof. Set $M_i := \dfrac{d}{N} |\psi_i\rangle\langle\psi_i|$. Then $M_i \succeq 0$ and $\sum_i M_i = \frac{d}{N} \sum_i |\psi_i\rangle\langle\psi_i| = \mathbb{1}$, so $\{M_i\}$ is a POVM.

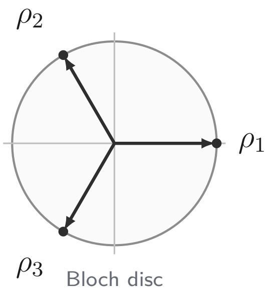
$$p_{\text{succ}}(\{M_i\}) = \frac{d}{N^2} \sum_{i=1}^{N} \operatorname{tr}(|\psi_i\rangle\langle\psi_i|\,|\psi_i\rangle\langle\psi_i|) = \frac{d}{N}.$$

The dual feasible point $Y = \frac{\mathbb{1}}{N}$ gives $\operatorname{tr}(Y) = \frac{d}{N}$. $\square$

The trine states

Let $\omega = e^{2\pi \mathrm{i}/3}$ and, for $i = 1, 2, 3$,

$$|\psi_i\rangle = \frac{|0\rangle + \omega^{i-1}|1\rangle}{\sqrt{2}}, \qquad \rho_i = |\psi_i\rangle\langle\psi_i|$$



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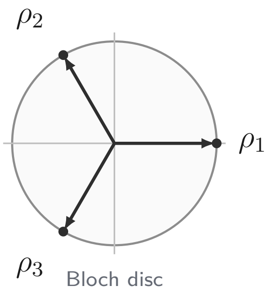
$$\frac{1}{3}\sum_{i=1}^{3}\rho_i = \frac{\mathbb{1}}{2}$$



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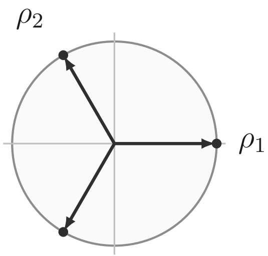
$$\frac{1}{3}\sum_{i=1}^{3} \rho_i = \frac{\mathbb{1}}{2}$$

$$M_i = \frac{2}{3}\rho_i, \qquad Y = \frac{\mathbb{1}}{3}, \qquad p^{\star}_{\text{succ}} = \frac{2}{3}$$

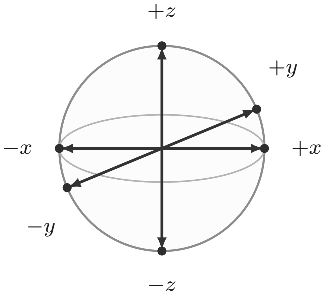


Bloch disc

Pauli eigenstates form an optimal octahedron

For $a \in \{x, y, z\}$ and $s \in \{+1, -1\}$, let

$$\rho_{a,s} = \frac{1}{2}(\mathbb{1} + s\sigma_a)$$



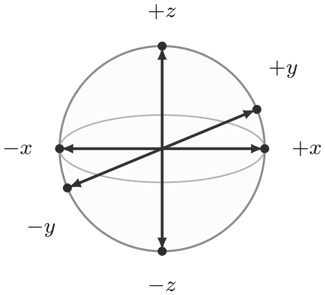
Bloch sphere

Pauli eigenstates form an optimal octahedron

For $a \in \{x, y, z\}$ and $s \in \{+1, -1\}$, let

$$\rho_{a,s} = \frac{1}{2}(\mathbb{1} + s\sigma_a)$$

$$\frac{1}{6} \sum_{a\in\{x,y,z\}} \sum_{s\in\{+1,-1\}} \rho_{a,s} = \frac{\mathbb{1}}{2}$$



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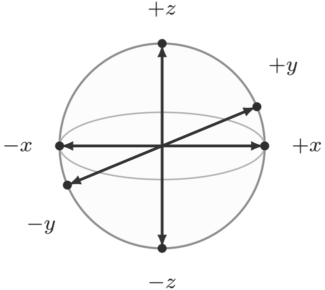
$$\rho_{a,s} = \frac{1}{2}(\mathbb{1} + s\sigma_a)$$

$$\frac{1}{6} \sum_{a\in\{x,y,z\}} \sum_{s\in\{+1,-1\}} \rho_{a,s} = \frac{\mathbb{1}}{2}$$

$$M_{a,s} = \frac{1}{3}\rho_{a,s}, \qquad Y = \frac{\mathbb{1}}{6}, \qquad p^\star_{\text{succ}} = \frac{1}{3}$$



Bloch sphere

8. Numerical solvers

From an exact model to approximate candidates

Numerical solvers

- There are *efficient* computational algorithms to solve SDPs.

Numerical solvers

- There are *efficient* computational algorithms to solve SDPs.
- For a well-posed SDP of manageable size, a solver can usually return approximate primal and dual candidates—or diagnose a problem.

Efficient means controlled scaling

An SDP algorithm receives

$$\text{matrix data} \quad + \quad \text{an accuracy request } \varepsilon$$

Efficient means controlled scaling

An SDP algorithm receives

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and returns approximate primal and dual candidates, objective values, and diagnostics.

Efficient means controlled scaling

An SDP algorithm receives

matrix data $\quad + \quad$ an accuracy request ε

and returns approximate primal and dual candidates, objective values, and diagnostics.

Under standard regularity and finite-precision assumptions, theoretical algorithms have polynomial scaling in the encoded input size and $\log(1/\varepsilon)$

The time complexity for solving an SDP

Given: $A = A^\dagger \in \mathcal{L}(\mathbb{C}^d)$, $\quad B = B^\dagger \in \mathcal{L}(\mathbb{C}^{d'})$

$\Lambda : \mathcal{L}(\mathbb{C}^d) \longrightarrow \mathcal{L}(\mathbb{C}^{d'})$ Hermiticity-preserving

$$\begin{aligned} \text{maximise} \quad & \operatorname{tr}(A\rho) \\ \text{subject to} \quad & \Lambda(\rho) = B, \qquad \rho \succeq 0 \end{aligned}$$

The time complexity for solving an SDP

Given: $A = A^\dagger \in \mathcal{L}(\mathbb{C}^d)$, $\quad B = B^\dagger \in \mathcal{L}(\mathbb{C}^{d'})$

$\Lambda : \mathcal{L}(\mathbb{C}^d) \longrightarrow \mathcal{L}(\mathbb{C}^{d'})$ Hermiticity-preserving

$$\begin{aligned} \text{maximise} \quad & \operatorname{tr}(A\rho) \\ \text{subject to} \quad & \Lambda(\rho) = B, \qquad \rho \succeq 0 \end{aligned}$$

Roughly speaking,

the time needed to solve this problem to precision ε grows polynomially in d, d', and $\log(1/\varepsilon)$.

Examples?

The primal SDP is one short function

```julia
using LinearAlgebra
using JuMP
import Clarabel

function state_discrimination(rho, p)
    N, d = length(rho), size(rho[1], 1)
    model = Model(Clarabel.Optimizer)
    M = [@variable(model, [1:d, 1:d] in HermitianPSDCone()) for i in 1:N]
    @constraint(model, sum(M) == I)
    @objective(
        model,
        Max,
        sum(p[i] * real(tr(rho[i] * M[i])) for i in 1:N),
    )
    optimize!(model)
    return objective_value(model), value.(M)
end
```

The dual is just as direct

```
function state_discrimination_dual(rho, p)
    N, d = length(rho), size(rho[1], 1)
    model = Model(Clarabel.Optimizer)
    @variable(model, Y[1:d, 1:d] in HermitianPSDCone())
    @constraint(
        model,
        [i = 1:N],
        Hermitian(Y - p[i] * rho[i]) in HermitianPSDCone(),
    )
    @objective(model, Min, real(tr(Y)))
    optimize!(model)
    return objective_value(model), value.(Y)
end
```

Discriminating $|0\rangle$ from $|+\rangle$ in a few lines

```
include("state_discrimination.jl")

ket0 = [1; 0]
ket_plus = [1; 1] / sqrt(2)

rho_1 = ket0 * ket0'
rho_2 = ket_plus * ket_plus'
p_1 = 1 / 2
p_2 = 1 / 2

rho = [rho_1, rho_2]
p = [p_1, p_2]

p_primal, M = state_discrimination(rho, p)
p_dual, Y = state_discrimination_dual(rho, p)

exact = (2 + sqrt(2)) / 4

println("rho      = ", rho)
println("p        = ", p)
println("primal   = ", p_primal)
println("dual     = ", p_dual)
println("expected = ", exact)
```

Live demonstration!

The numerical outcome

NUMERICAL

primal	0.8535533694
dual	0.8535533898
expected	0.8535533906

EXACT

$$p^{\star}_{\text{succ}} = \frac{2+\sqrt{2}}{4}$$

$$Y^{\star} = \frac{1}{4}\left(\rho_1 + \rho_2 + \frac{\mathbb{1}}{\sqrt{2}}\right)$$

$$M_1 = |m_1\rangle\langle m_1| , \qquad M_2 = |m_2\rangle\langle m_2|$$

Discriminating the six Pauli eigenstates

```julia
include("state_discrimination.jl")
sigma_x = [0 1; 1 0]
sigma_y = [0 -im; im 0]
sigma_z = [1 0; 0 -1]

rho = [Matrix((I + s * sigma) / 2)
       for sigma in (sigma_x, sigma_y, sigma_z) for s in (1, -1)]
p = fill(1 / 6, 6)

p_primal, M = state_discrimination(rho, p)
p_dual, Y = state_discrimination_dual(rho, p)

exact = 1 / 3

println("rho      = ", rho)
println("p        = ", p)
println("primal   = ", p_primal)
println("dual     = ", p_dual)
println("expected = ", exact)
```

The numerical outcome

NUMERICAL

primal	0.3333333325
dual	0.3333333330
expected	0.3333333333

The optimal POVM is not unique.

EXACT

$$\color{green}{M_{a,s}} = \frac{1}{3}\rho_{a,s}, \qquad \color{magenta}{Y} = \frac{1}{6}\mathbb{1}$$

$$p^{\star}_{\mathrm{succ}} = \frac{1}{3}$$

One exact optimal pair, independent of the solver output

Solving SDPs with computers

Language

Julia

Open source; fast.

MATLAB

Proprietary; fast.

Python

Open source; broad ecosystem; model construction can add overhead.

Solving SDPs with computers

Language	Modelling layer
Julia Open source; fast.	**JuMP** Open source.
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Python Open source; broad ecosystem; model construction can add overhead.	**CVXPY** Open source.

Solvers

Clarabel	**SCS**	**MOSEK**
Open source; modern.	Open source; fast; modest accuracy.	Proprietary; free for academics; fast and robust.

Quantum information libraries

Quantum-information libraries can simplify common tasks:

- Partial traces, partial transposition, and other convenient functions

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Julia	MATLAB	Python
Ket.jl	**QETLAB**	$\vert$**toqito**$\rangle$

This is not an exhaustive list—only a few libraries I have used and liked.

Floating-point arithmetic?

Floating-point arithmetic is powerful and reliable.

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But this is not a mathematical proof!

9. Computer-assisted proofs

From numerical structure to rigorous bounds

Do you trust your computer?

Sometimes yes, sometimes no.

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Sometimes yes, sometimes no.

Floating-point numbers are convenient.
But do you trust them?

What is a floating-point number?

As in scientific notation, a normal binary floating-point number has the form

$$\underbrace{(-1)^s}_{\text{sign}} \times \underbrace{(1.b_1 b_2 \ldots b_k)_2}_{\text{significand}} \times \underbrace{2^e}_{\text{exponent}}$$

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$$\underbrace{(-1)^s}_{\text{sign}} \times \underbrace{(1.b_1 b_2 \ldots b_k)_2}_{\text{significand}} \times \underbrace{2^e}_{\text{exponent}}$$

Only finitely many values can be stored exactly.

Most inputs are rounded to a nearby representable number.

Distinct numbers can share one Float64

EXACT

$$x = 9\,007\,199\,254\,740\,992, \qquad y = 9\,007\,199\,254\,740\,993$$

$$x \neq y$$

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NUMERICAL

$$\texttt{Float64}(x) = \texttt{Float64}(y) = 9\,007\,199\,254\,740\,992$$

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A finite floating-point format cannot represent every real number distinctly

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Live demonstration time!

Floating-point addition is not associative

In exact real arithmetic, addition is associative:

$$(a + b) + c = a + (b + c)$$

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$$a+(b+c) = 0.0$$

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$$a + (b + c) = 0.0$$

Live demonstration time!

The solver returns primal and dual candidates

$$p_1 = p_2 = \frac{1}{2}, \qquad \rho_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \qquad \rho_2 = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

Primal solver output

Dual solver output

$$p^{\text{num}}_{\text{primal}} = 0.8535533694$$

$$p^{\text{num}}_{\text{dual}} = 0.8535533898$$

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Primal solver output

$$p_{\text{primal}}^{\text{num}} = 0.8535533694$$

$$M_1^{\text{num}} \approx \begin{pmatrix} 0.853551 & -0.353555 \\ -0.353555 & 0.146449 \end{pmatrix}$$

$$M_2^{\text{num}} \approx \begin{pmatrix} 0.146449 & 0.353555 \\ 0.353555 & 0.853551 \end{pmatrix}$$

Dual solver output

$$p_{\text{dual}}^{\text{num}} = 0.8535533898$$

$$Y^{\text{num}} \approx \begin{pmatrix} 0.551772 & 0.124996 \\ 0.124996 & 0.301781 \end{pmatrix}$$

The variables are part of the numerical output too.

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Dual solver output

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The variables are part of the numerical output too.

Can these decimal candidates lead to an exact proof?

A recipe for a dual certificate

For the dual:

1. Round or reconstruct an exact Hermitian candidate:

$$Y^{\mathrm{num}} \rightsquigarrow \widetilde{Y} = \widetilde{Y}^{\dagger}.$$

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$$\lambda_{\min}\big(\widetilde{Y} - p_i \rho_i\big) \geq -\varepsilon \quad \forall i.$$

Then

$$Y^{\mathrm{cert}} := \widetilde{Y} + \varepsilon \mathbb{1} \succeq p_i \rho_i \quad \forall i.$$

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   Then

$$Y^{\mathrm{cert}} := \widetilde{Y} + \varepsilon \mathbb{1} \succeq p_i \rho_i \quad \forall i.$$

3. Evaluate $\mathrm{tr}(Y^{\mathrm{cert}})$ exactly: this is a rigorous upper bound.

An exact analytical proof from numerics

- Reconstruct an exact matrix from the decimal pattern:[1]

$$Y^{\text{exact}} := \frac{1}{8}\begin{pmatrix} 3+\sqrt{2} & 1 \\ 1 & 1+\sqrt{2} \end{pmatrix}.$$

[1] Of course, this is not always possible. But sometimes, yes! It is worth doing a few checks.

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- Verify both inequalities by exact rank-one factorisations:

$$Y^{\text{exact}} - \frac{1}{2}\rho_1 = \frac{1}{2\sqrt{2}}\,|m_2\rangle\langle m_2| \succeq 0, \qquad Y^{\text{exact}} - \frac{1}{2}\rho_2 = \frac{1}{2\sqrt{2}}\,|m_1\rangle\langle m_1| \succeq 0.$$

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- Evaluate the dual objective exactly:

$$p^\star_{\text{succ}} \leq p^\star_{\text{upper}} := \operatorname{tr}\left(Y^{\text{exact}}\right) = \frac{(3+\sqrt{2})+(1+\sqrt{2})}{8} = \frac{2+\sqrt{2}}{4}.$$

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$$R := \mathbb{1} - \sum_j \widetilde{M}_j, \qquad \widehat{M}_i := \widetilde{M}_i + \frac{R}{N}, \qquad \sum_i \widehat{M}_i = \mathbb{1}.$$

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$$M_i^{\mathrm{cert}} := (1-\eta)\widehat{M}_i + \eta\frac{\mathbb{1}}{N} \succeq 0 \quad \forall i.$$
4. Evaluate the primal objective exactly to obtain a rigorous lower bound:
$$\sum_i p_i \operatorname{tr}\Big(\rho_i M_i^{\mathrm{cert}}\Big) \leq p_{\mathrm{succ}}^{\star}.$$

Certify the primal candidate

- Reconstruct exact matrices from the decimal pattern:[2]

$$M_1^{\text{exact}} := \frac{1}{4}\begin{pmatrix} 2+\sqrt{2} & -\sqrt{2} \\ -\sqrt{2} & 2-\sqrt{2} \end{pmatrix}, \qquad M_2^{\text{exact}} := \frac{1}{4}\begin{pmatrix} 2-\sqrt{2} & \sqrt{2} \\ \sqrt{2} & 2+\sqrt{2} \end{pmatrix}.$$

[2] Again, not always possible. But sometimes it works!

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- Verify the constraints exactly:

$$M_i^{\text{exact}} = |m_i\rangle\langle m_i| \succeq 0 \qquad (i = 1, 2),$$
$$M_1^{\text{exact}} + M_2^{\text{exact}} = \frac{1}{4}\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} = \mathbb{1}.$$

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- Evaluate the primal objective:

$$\begin{aligned} p_{\text{lower}}^{\star} &:= \frac{1}{2}\operatorname{tr}\big(\rho_1 M_1^{\text{exact}}\big) + \frac{1}{2}\operatorname{tr}\big(\rho_2 M_2^{\text{exact}}\big) \\ &= \frac{2+\sqrt{2}}{4} = p_{\text{upper}}^{\star}, \end{aligned}$$

$$p_{\text{lower}}^{\star} \le p_{\text{succ}}^{\star} \le p_{\text{upper}}^{\star} \quad \Longrightarrow \quad \boxed{p_{\text{succ}}^{\star} = \frac{2+\sqrt{2}}{4}}.$$

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10. Outlook

What the certificate viewpoint unlocks

Lecture 2 in brief

- The dual problem can provide good (and tight) upper bounds.

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- Slater conditions ensure strong duality.
- There are efficient numerical methods.
- Numerical solutions can sometimes be turned into computer-assisted proofs.

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- State discrimination is **clearly an SDP**. For some problems, this is not at all obvious.
- Sometimes we must **“massage” the formulation** to recognise its SDP structure.
- If all else fails, **SDP hierarchies** may still provide systematic upper and/or lower bounds.
- This should make more sense during Jessica’s talk.

SDP in quantum information

- entanglement and steering
- measurement compatibility
- Bell nonlocality and correlations in networks
- quantum marginal problem
- channel discrimination
- quantum channel capacities
- quantum metrology
- higher-order computing, indefinite causality
- ...

Want more?

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Thank you! 🌠