Composable simultaneous purification: when all communication scenarios reduce to spatial correlations

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arXiv:2601.05158 (2026)

Outline

- Bell nonlocality (spacial correlations)

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- Prepare-transform-and-measure (temporal correlations)

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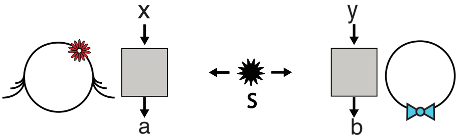
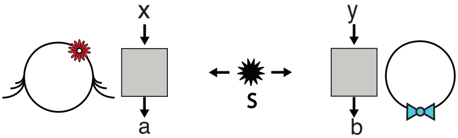
- Two-way communication? Indefinite casual order? GPTs?

Motivation

Bipartite Bell nonlocality

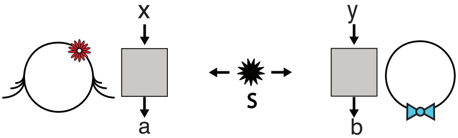


Bipartite Bell nonlocality



$$p(ab|xy) = \mathrm{tr}\big(\rho_{AB} A_{a|x} \otimes B_{b|y}\big)$$

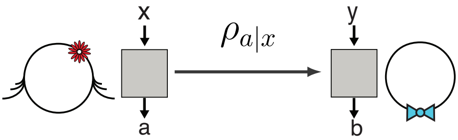
Bipartite Bell nonlocality



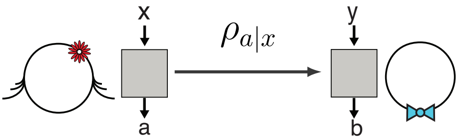
$$p(ab|xy) = \mathrm{tr}\left(\rho_{AB} A_{a|x} \otimes B_{b|y}\right)$$

$$p(ab|xy) \neq \sum_{\lambda} \pi(\lambda) p_A(a|x,\lambda) p_B(b|y,\lambda)$$

Prepare-and-measure scenario (temporal correlations)



Prepare-and-measure scenario (temporal correlations)



$$p(ab|xy) = \mathrm{tr}\big(\rho_{a|x} B_{b|y}\big)$$

Spatial vs temporal correlations

Spatial correlations

can always be obtained via

temporal correlations

Spatial vs temporal correlations

$$\boxed{\rho_{AB},\ \{A_{a|x}\},\ \{B_{b|y}\}}$$

spatial strategy

$$\boxed{\{\rho_{a|x}\},\ \{B_{b|y}\}}$$

temporal assemblage

$$\rho_{a|x} := \mathrm{tr}_A\big[(A_{a|x} \otimes \mathbb{1}_B)\rho_{AB}\big]$$

Spatial vs temporal correlations

$$\rho_{AB},\ \{A_{a|x}\},\ \{B_{b|y}\}$$

spatial strategy

$$\{\rho_{a|x}\},\ \{B_{b|y}\}$$

temporal assemblage

$$\rho_{a|x} := \mathrm{tr}_A\big[(A_{a|x} \otimes \mathbb{1}_B)\rho_{AB}\big]$$

$$\boxed{\mathrm{tr}\big(\rho_{a|x} B_{b|y}\big) = \mathrm{tr}\big[\rho_{AB}\left(A_{a|x} \otimes B_{b|y}\right)\big]}$$

Every spatial behaviour is therefore a temporal behaviour.

Spatial vs temporal correlations 2

temporal correlations

cannot be obtained via

spatial correlations

Spatial vs temporal correlations 2

In a temporal scenario, Alice can simply send her input x to Bob.

Spatial vs temporal correlations 2

In a temporal scenario, Alice can simply send her input x to Bob.

This produces a signalling behaviour, which is impossible in a non-signalling spatial scenario.

Spatial vs temporal correlations 2

Alice

$\rho_{a|x} = \delta_{a,0}\,|x\rangle\langle x|$

$\xrightarrow{\text{state carrying } x}$

Bob

$B_{b|y} = |b\rangle\langle b|$

Spatial

$$\underbrace{\sum_a \rho_{a|x} \;=\; \rho_B}_{\text{independent of } x}$$

$\neq$

This example

$$\underbrace{\sum_a \rho_{a|x} \;=\; |x\rangle\langle x|}_{\text{depends on } x}$$

$$p(a,b|x,y) = \delta_{a,0}\delta_{b,x} \quad \Longrightarrow \quad b = x$$

Alice can signal to Bob; no spatial Bell strategy can.

The bipartite question

When does a temporal behaviour admit a spatial realisation?

Temporal

$$p(ab|xy) = \operatorname{tr}\left(\rho_{a|x} B_{b|y}\right)$$

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$$p(ab|xy) = \mathrm{tr}\left[\rho_{AB}\left(A_{a|x} \otimes B_{b|y}\right)\right]$$

The bipartite question

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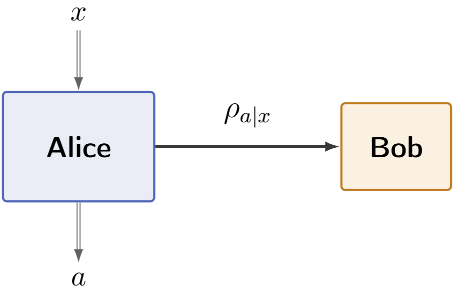
Spatial

$$p(ab|xy) = \mathrm{tr}\left[\rho_{AB}\left(A_{a|x} \otimes B_{b|y}\right)\right]$$

For which state assemblages $\{\rho_{a|x}\}$ is this possible for every measurement performed by Bob?

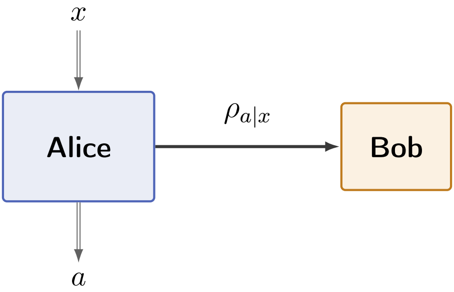
Proof method

Non-signalling assemblages



After Alice's outcome a is forgotten,
Bob's state contains no information about x.

Non-signalling assemblages



After Alice's outcome a is forgotten,
Bob's state contains no information about x.

$$\sum_a \rho_{a|x} = \rho \qquad \forall x$$

The same marginal state for every setting.

The SGHJW theorem

Schrödinger–Gisin–Hughston–Jozsa–Wootters–∞

Non-signalling assemblage

$$\sum_a \rho_{a|x} \quad = \quad \rho_B \qquad \forall x$$

The SGHJW theorem

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Non-signalling assemblage

$$\sum_a \rho_{a|x} \quad = \quad \rho_B \qquad \forall x$$

$$\Longleftrightarrow$$

Spatial realisation

There exist a pure state $|\psi\rangle_{AB}$
and POVMs $\{A_{a|x}\}_a$ such that

$$\rho_{a|x} \quad = \quad \mathrm{Tr}_A\Big[\big(A_{a|x} \otimes \mathbb{1}_B\big)\, |\psi\rangle\langle\psi|_{AB}\Big]\,.$$

The SGHJW theorem

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$$\rho_{a|x} = \mathrm{Tr}_A\left[\left(A_{a|x} \otimes \mathbb{1}_B\right) |\psi\rangle\langle\psi|_{AB}\right].$$

Purification

$$|\psi\rangle_{AB} = \sum_i \sqrt{\lambda_i}\, |i\rangle_A\, |i\rangle_B$$

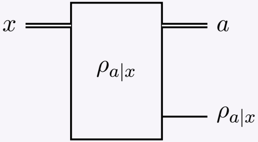
Alice's POVMs

$$A_{a|x} = \rho_B^{-1/2} \rho_{a|x}^T \rho_B^{-1/2}$$

(the inverse is taken on the support of ρ_B)

SGHJW in circuit notation

$$\sum_a \rho_{a|x} = \rho_B \quad \forall x \quad \Longleftrightarrow \quad \exists\, |\psi\rangle_{AB}\,,\ \{A_{a|x}\}_a$$



State assemblage

SGHJW in circuit notation

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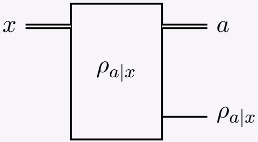
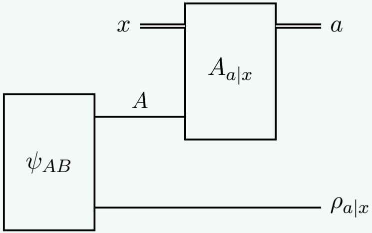


State assemblage

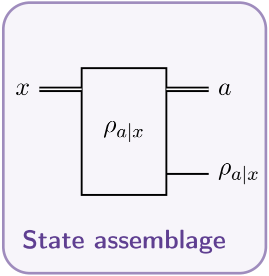
$=$



Purification + local measurement

A realisation for every state assemblage

No non-signalling assumption is required.



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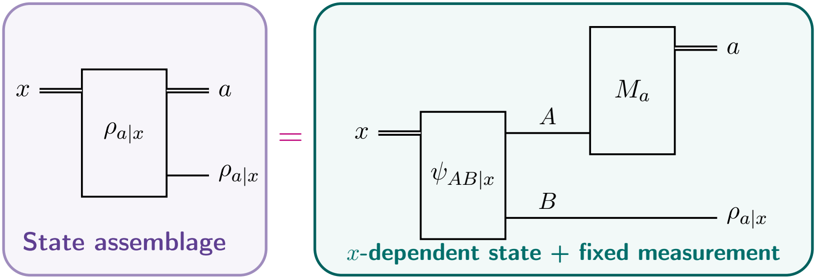

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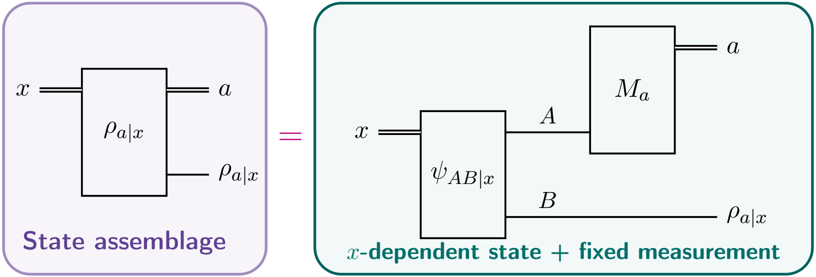
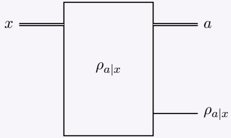


$$\rho_{a|x} \quad = \quad \mathrm{Tr}_A\left[(M_a \otimes \mathbb{1}_B)\psi_{AB|x}\right]$$

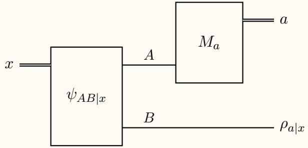
for every assemblage $\{\rho_{a|x}\}_{a,x}$.

Three state-assemblage circuits

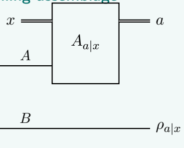
State assemblage



Every assemblage

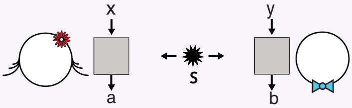


Non-signalling assemblage



Notation and diagrammatic conventions

Bell: top to bottom



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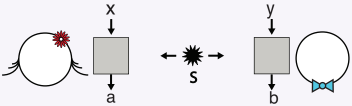
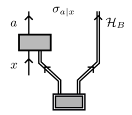


QPL: bottom to top



(a)

Zjawin et al., *Activation of post-quantumness in bipartite generalised EPR scenarios*, PRA **110**, 042212 (2024)

Notation and diagrammatic conventions

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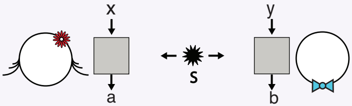

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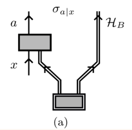
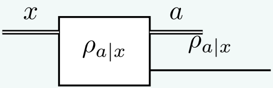


Quantum circuits



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Notation and diagrammatic conventions

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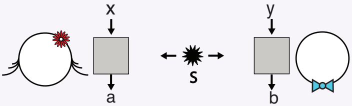
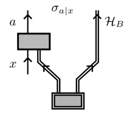


QPL: bottom to top



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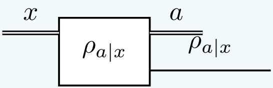


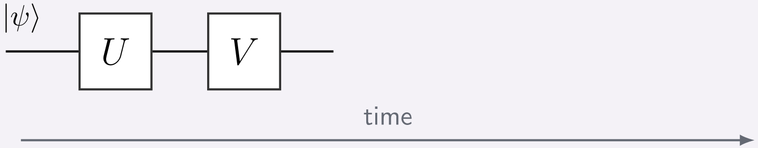
Mathematical composition

$$f \circ g$$

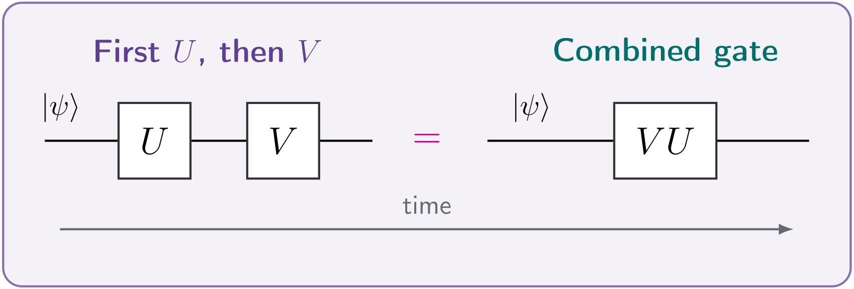
Zjawin et al., *Activation of post-quantumness in bipartite generalised EPR scenarios*, PRA 110, 042212 (2024)

Notation and diagrammatic conventions

First U, then V



Notation and diagrammatic conventions



$$V(U|\psi\rangle) \quad = \quad (VU)|\psi\rangle$$

A small digression

Circuits from right to left?

A small digression

Circuits from right to left?

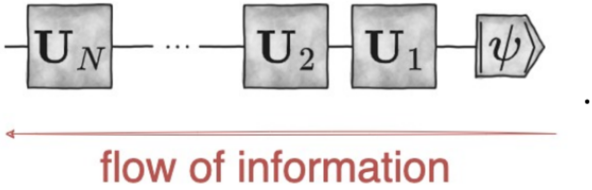
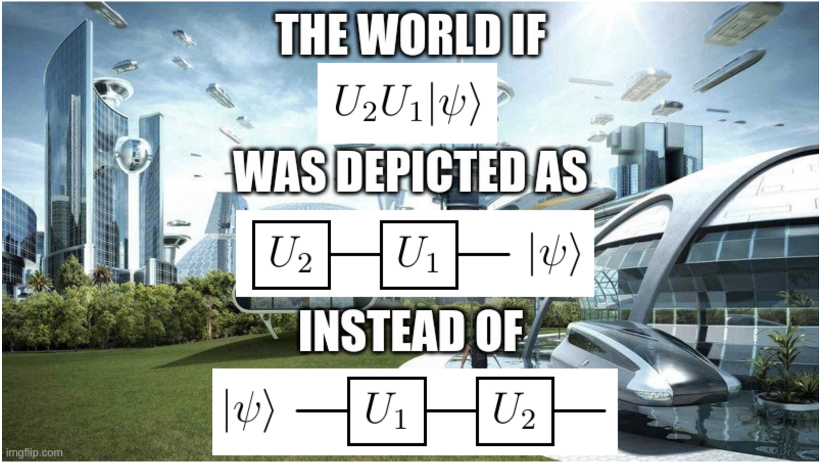
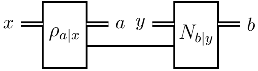


Figure from Richard Kueng's quantum-computing book.


THE WORLD IF
$U_2 U_1 |\psi\rangle$
WAS DEPICTED AS
U_2 — U_1 — $|\psi\rangle$
INSTEAD OF
$|\psi\rangle$ — U_1 — U_2 —
imgflip.com

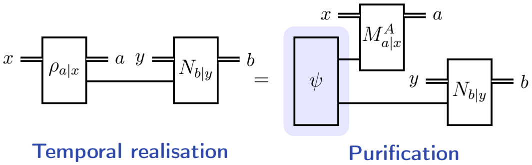
Now, back to our problem

NS temporal scenarios and spatial scenarios (bipartite case)



Temporal realisation

NS temporal scenarios and spatial scenarios (bipartite case)



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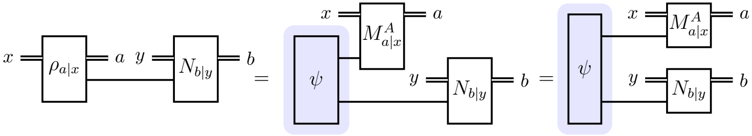

Temporal realisation **Purification** **Spatial realisation**

The classical bipartite question

When does a

quantum temporal correlation

admit a classical spatial

realisation (Bell-local)?

The classical bipartite question

When does a
quantum temporal correlation
admit a classical spatial realisation (Bell-local)?

$\Downarrow$

We delay the classical input!

Unsteerable state assemblages

$$\rho_{a|x} \quad = \quad \sum_{\lambda} p(\lambda)\, p(a|x,\lambda)\, \rho_{\lambda}$$

Wiseman, Jones, and Doherty, *Steering, Entanglement, Nonlocality, and the EPR Paradox*, PRL **98**, 140402 (2007).

Unsteerable state assemblages

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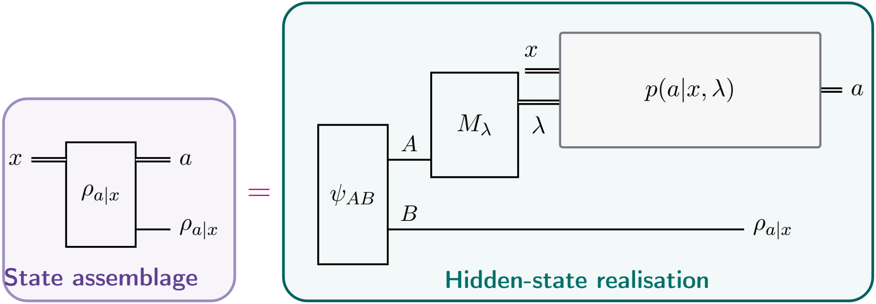
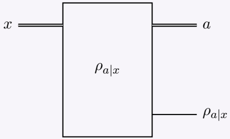


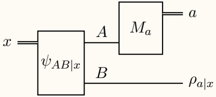
Wiseman, Jones, and Doherty, *Steering, Entanglement, Nonlocality, and the EPR Paradox*, PRL **98**, 140402 (2007).

Four state-assemblage circuits

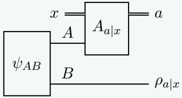
State assemblage



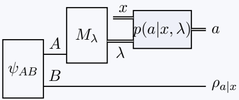
Every assemblage



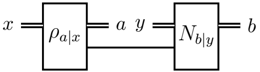
Non-signalling assemblage



Hidden-state realisation

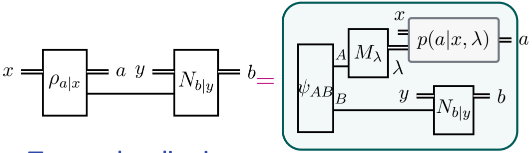


Unsteerable temporal correlations are Bell-local



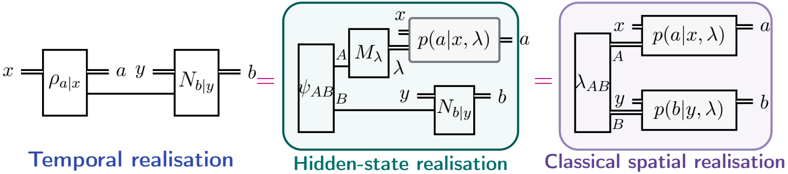
Temporal realisation

Unsteerable temporal correlations are Bell-local



Temporal realisation **Hidden-state realisation**

Unsteerable temporal correlations are Bell-local



Fine, *Hidden Variables, Joint Probability, and the Bell Inequalities*, PRL **48**, 291 (1982).

Tripartite Bell nonlocality

The bipartite construction works!

Wright,S Farkas, *Invertible Map between Bell Nonlocal and Contextuality Scenarios*, PRL (2023).

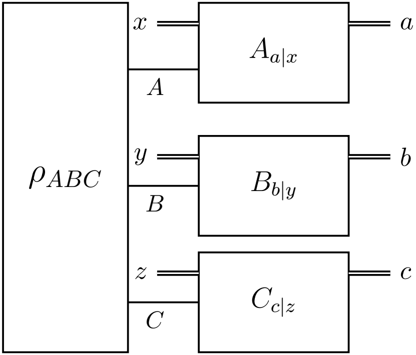
Tripartite Bell nonlocality

The bipartite construction works!

Does it extend to three parties?

Wright,S Farkas, *Invertible Map between Bell Nonlocal and Contextuality Scenarios*, PRL (2023).

Tripartite Bell nonlocality



$$p(abc|xyz) = \mathrm{tr}\left[\rho_{ABC}\left(A_{a|x} \otimes B_{b|y} \otimes C_{c|z}\right)\right]$$

Tripartite Bell nonlocality NOW

Bipartite

NS assemblage

$\Downarrow$

quantum realisation

SGHJW ✓

Tripartite Bell nonlocality NOW

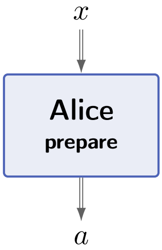
Bipartite	**Tripartite**
NS assemblage	NS assemblage
$\Downarrow$	$\not\Rightarrow$
quantum realisation	quantum realisation
SGHJW $\checkmark$	**post-quantum steering**

"Lastly, one might attempt to generalise our map to multipartite Bell scenarios."
"Whether this map also preserves quantumness remains unknown, with the existence of post-quantum steering [17] posing an obstacle to generalising our argument."
Wright, Farkas (2023)

A. B. Sainz et al., *Postquantum steering*, Phys. Rev. Lett. **115**, 190403 (2015).

Tripartite Bell nonlocality

Prepare–transform–and–measure



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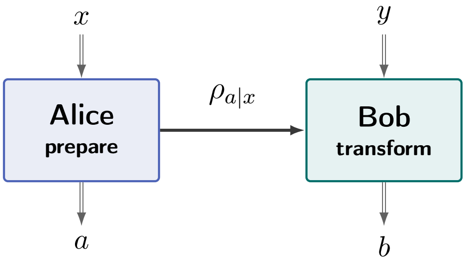

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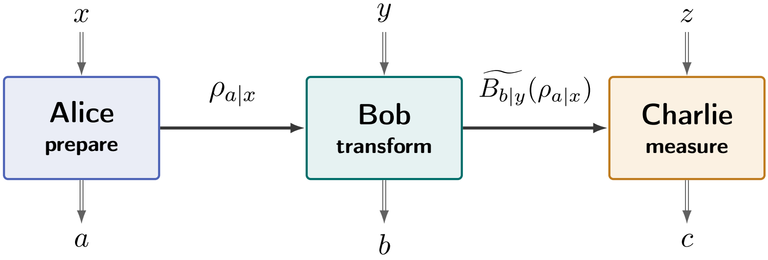

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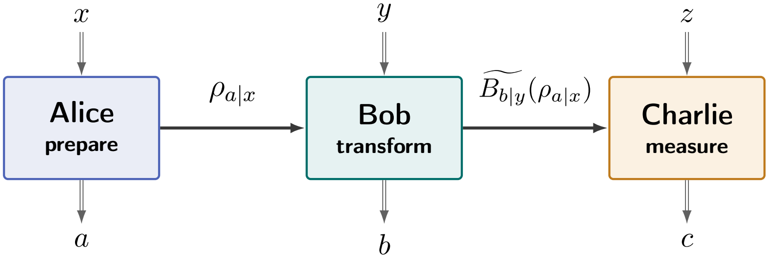

$$p(abc|xyz) = \mathrm{tr}\left(\widetilde{B_{b|y}}(\rho_{a|x}) C_{c|z}\right)$$

Quantum instrument

For every setting y, the family $\{\widetilde{B_{b|y}}\}_b$ is a quantum instrument when

Instrument element is CP

$$\widetilde{B_{b|y}} \in \mathrm{CP}$$

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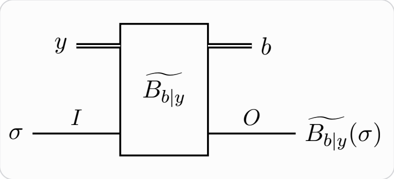
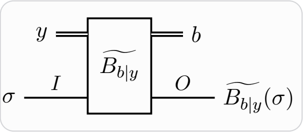


Stinespring dilation

Every quantum instrument admits an **isometric realisation**.



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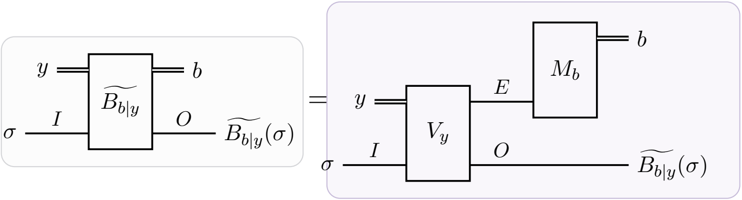

NS quantum instrument

A set of instruments $\{\widetilde{B_{b|y}}\}_{b,y}$ is **no-signalling** when

Definition

$$\sum_b \widetilde{B_{b|y}} = \widetilde{B} \qquad \forall\, y$$

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Changing y does not change the unconditional channel.

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Theorem

A family $\{\widetilde{B_{b|y}}\}_{b,y}$ is no-signalling if and only if it admits a common dilation.

No-signalling

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No-signalling

$$\sum_b \widetilde{B_{b|y}} = \widetilde{B}$$

$$\forall\, y$$

$\Longleftrightarrow$

Common dilation

$\exists\, \widetilde{V},\ \{B_{b|y}\}_b$ such that

$$\widetilde{B_{b|y}}(\sigma) = \mathrm{Tr}_A\Big[\big(B_{b|y} \otimes \mathbb{1}_O\big)\,\widetilde{V}(\sigma)\Big]$$

M. Raginsky, *Radon–Nikodym derivatives of quantum operations*, J. Math. Phys. **44**, 5003–5020 (2003).

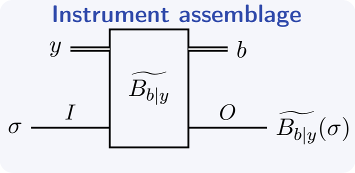
V. P. Belavkin and P. Staszewski, *A Radon–Nikodym theorem for completely positive maps*, Rep. Math. Phys. **24**, 49–55 (1986).

R. Uola, F. Lever, O. Gühne, and J.-P. Pellonpää, *Unified picture for spatial, temporal, and channel steering*, Phys. Rev. A **97**, 032301 (2018).

NS instruments

Theorem

If the instrument assemblage is no-signalling, then



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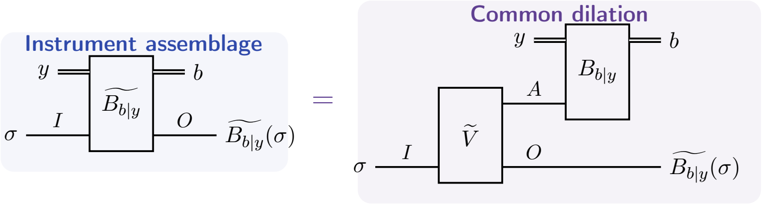
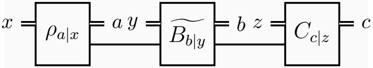


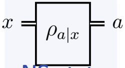
The dilation $\widetilde{V}$ is independent of the setting y.

From temporal components to a spatial realisation



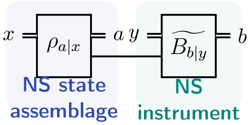
Temporal realisation

From temporal components to a spatial realisation

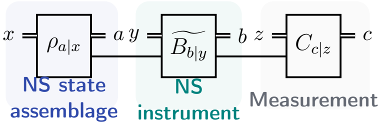


NS state assemblage

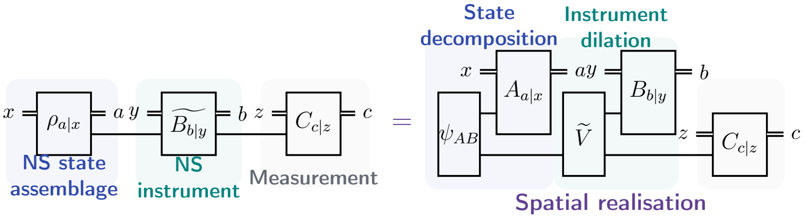
From temporal components to a spatial realisation



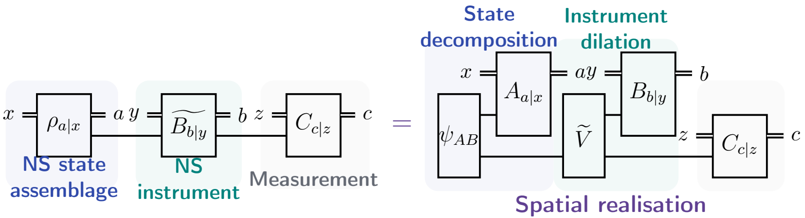
From temporal components to a spatial realisation

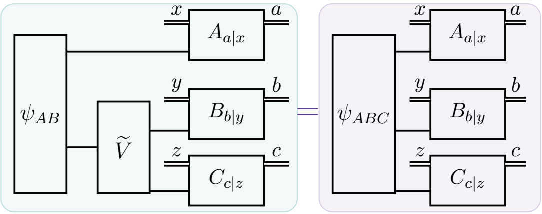


From temporal components to a spatial realisation



From temporal components to a spatial realisation





Two NS constraints recover spatial correlations

Prepare–transform–and–measure correlation

$$p(a, b, c|x, y, z) = \mathrm{Tr}\left[C_{c|z}\, \widetilde{B_{b|y}}\left(\rho_{a|x}\right)\right]$$

NS preparation

$$\sum_a \rho_{a|x} = \rho \qquad \forall\, x$$

NS transformation

$$\sum_b \widetilde{B_{b|y}} = \widetilde{B} \qquad \forall\, y$$

Baroni et al., arXiv:2601.05158 (2026)

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$$\Downarrow$$

Tripartite spatial realisation

$$p(a,b,c|x,y,z) = \mathrm{Tr}\left[\left(A_{a|x}\otimes B_{b|y}\otimes C_{c|z}\right)\psi_{ABC}\right]$$

$$\mathcal{Q}^{\mathrm{NS}}_{\mathrm{PTM}} = \mathcal{Q}^{(3)}_{\mathrm{spatial}}$$

Baroni et al., arXiv:2601.05158 (2026)

The classical case: unsteerable instruments

Unsteerable instrument assemblages

An instrument assemblage $\{\widetilde{B_{b|y}}\}_{b,y}$ is **unsteerable** when it admits the decomposition

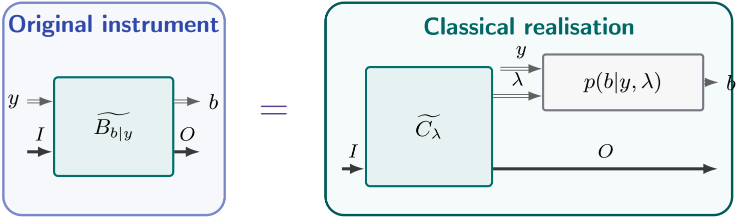
$$\widetilde{B_{b|y}} \quad = \quad \sum_{\lambda} p(\lambda)\, p(b|y,\lambda)\, \widetilde{C_\lambda}$$

where $p(\lambda)$ is a probability distribution, $p(b|y,\lambda)$ a response function, and each $\widetilde{C_\lambda}$ a channel.

Piani, JOSA B **32**, A1 (2015); Uola et al., PRA **97**, 032301 (2018); Baroni et al., arXiv:2601.05158 (2026)

The classical case: unsteerable instruments

Unsteerable $\iff$ $\widetilde{B_{b|y}} = \sum_\lambda p(\lambda)\, p(b|y,\lambda)\, \widetilde{C_\lambda}$



The setting y changes only the classical response;
λ selects the channel independently of y.

Piani, JOSA B 32, A1 (2015); Uola et al., PRA **97**, 032301 (2018); Baroni et al., arXiv:2601.05158 (2026)

Unsteerable instruments lead to classical behaviours

Prepare–transform–and–measure behaviour

$$p(a,b,c|x,y,z) = \mathrm{Tr}\left[C_{c|z}\,\widetilde{B_{b|y}}\left(\rho_{a|x}\right)\right]$$

Unsteerable preparation

$$\rho_{a|x} = \sum_{\mu} p(\mu)\,p(a|x,\mu)\,\rho_{\mu}$$

Unsteerable instrument

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$$\Downarrow$$

Classical spatial realisation

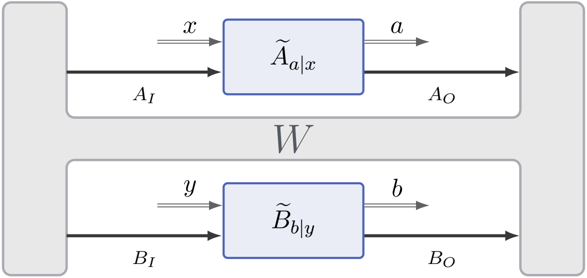
$$p(a,b,c|x,y,z) = \sum_{\mu,\lambda} p(\mu)p(\lambda)\,p(a|x,\mu)\,p(b|y,\lambda)\,p(c|z,\mu,\lambda)$$

$$p(c|z,\mu,\lambda) \quad := \quad \mathrm{Tr}\left[C_{c|z}\,\widetilde{C_{\lambda}}(\rho_{\mu})\right]$$

Baroni et al., arXiv:2601.05158 (2026)

Process matrices and indefinite causal order

A process matrix does not presuppose $A \prec B$ or $B \prec A$.



Generalised Born rule

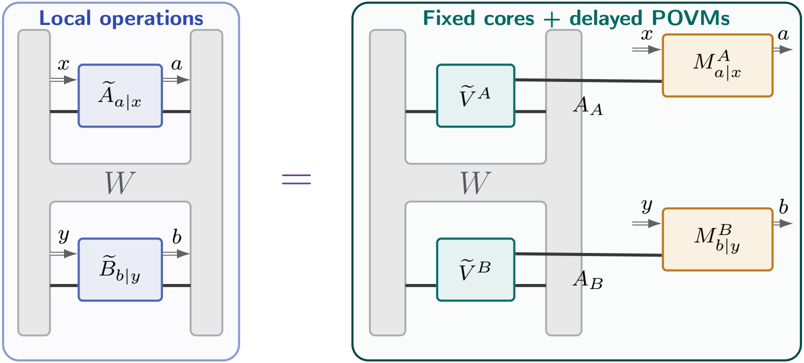
$$p(a,b|x,y) \quad = \quad \mathrm{Tr}\left[W\left(A_{a|x} \otimes B_{b|y}\right)\right]$$

$A_{a|x}, B_{b|y}$ are Choi operators of $\widetilde{A_{a|x}}, \widetilde{B_{b|y}}$;

Oreshkov, Costa, Brukner, Nat. Commun. 3, 1092 (2012).

Non-signalling local operations become spatial

$$\sum_a \widetilde{A_{a|x}} = \widetilde{A} \quad \forall x \qquad \text{and} \qquad \sum_b \widetilde{B_{b|y}} = \widetilde{B} \quad \forall y$$



Spatial realisation $p(a,b|x,y) = \mathrm{Tr}\left[\rho_{A_A A_B}\left(M^A_{a|x} \otimes M^B_{b|y}\right)\right]$

Baroni et al., arXiv:2601.05158 (2026)

Back-and-forth quantum communication

What if quantum systems are exchanged over many rounds?

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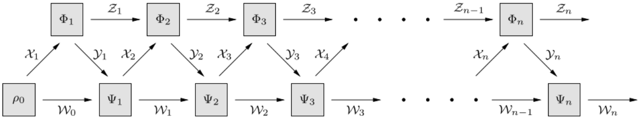


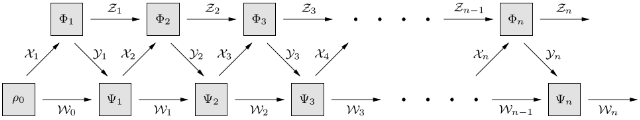
Figure 2: An interaction between an n-turn strategy and co-strategy.

Interactive games
players alternate quantum messages

Gutoski, Watrous, STOC 2007

Back-and-forth quantum communication

What if quantum systems are exchanged over many rounds?



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Channels with memory
memory links
successive uses

Kretschmann, Werner, PRA 72 (2005)

Back-and-forth quantum communication

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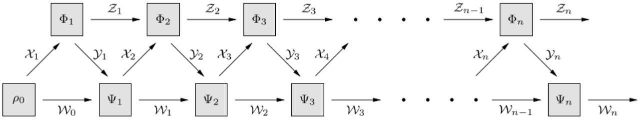

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Quantum combs
open slots for each round

Chiribella et al., PRL 101 (2008)

Back-and-forth quantum communication

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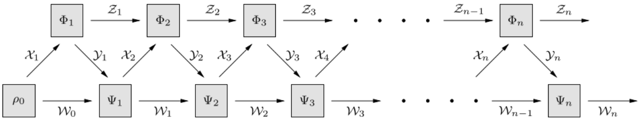

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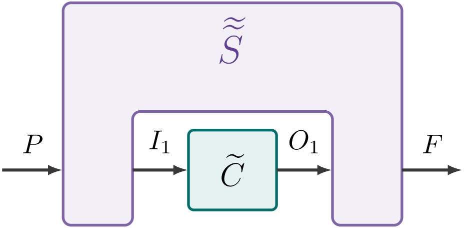
Back-and-forth communication is formalised by **superchannels**.

Quantum superchannels

A superchannel transforms a channel into another channel.



$$\widetilde{C}_{I_1 \to O_1} \longmapsto \widetilde{\widetilde{S}}(\widetilde{C})_{P \to F}$$

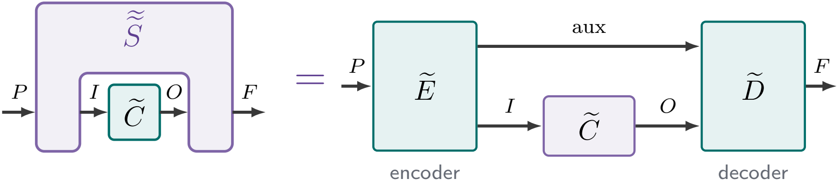
channels $\longmapsto$ channels

The open slot accepts an arbitrary channel $\widetilde{C}$; the surrounding higher-order map $\widetilde{\widetilde{S}}$ has no classical setting or outcome wires.

Chiribella, D'Ariano, Perinotti, Phys. Rev. Lett. **101**, 060401 (2008).

Circuit realisation of a superchannel

Every superchannel is an encoder–memory–decoder circuit.

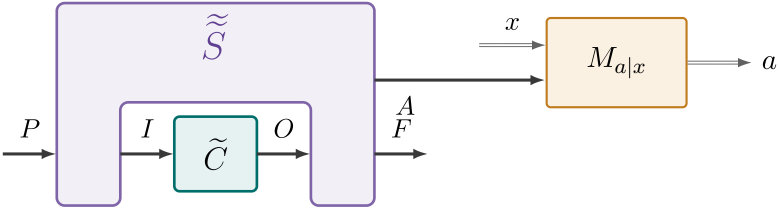


$$\widetilde{\widetilde{S}}(\widetilde{C}) = \widetilde{D} \circ \left(\widetilde{C} \otimes \mathrm{id}_{\mathrm{aux}}\right) \circ \widetilde{E} \qquad \widetilde{E} : P \to I \otimes \mathrm{aux},\ \widetilde{D} : O \otimes \mathrm{aux} \to F$$

Chiribella, D'Ariano, Perinotti, Phys. Rev. A **80**, 022339 (2009).

Quantum superinstruments

A superinstrument is a superchannel followed by a measurement.



superinstrument element $\widetilde{\widetilde{S_{a|x}}}$

$$\widetilde{\widetilde{S_{a|x}}}(\widetilde{C}) \;=\; \mathrm{Tr}_A\Big[\big(M_{a|x}\otimes \mathbb{1}_F\big)\widetilde{\widetilde{S}}(\widetilde{C})\Big]$$

$$\sum_a M_{a|x} \;=\; \mathbb{1}_A \quad\Longrightarrow\quad \sum_a \widetilde{\widetilde{S_{a|x}}} \;=\; \widetilde{\widetilde{S}}$$

NS quantum superinstrument

A superinstrument assemblage $\{\widetilde{\widetilde{S_{a|x}}}\}_{a,x}$ is **no-signalling** when

Definition

$$\sum_a \widetilde{\widetilde{S_{a|x}}} = \widetilde{\widetilde{S}} \qquad \forall\, x$$

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Definition

$$\sum_a \widetilde{\widetilde{S_{a|x}}} = \widetilde{\widetilde{S}} \qquad \forall\, x$$

Changing x does not change the unconditional superchannel.

Each $\widetilde{\widetilde{S_{a|x}}}$ probabilistically transforms an inserted channel $\widetilde{C}$.

Chiribella, D'Ariano, Perinotti, Phys. Rev. A **80**, 022339 (2009).

NS quantum superinstrument

Theorem

$\{\widetilde{\widetilde{S_{a|x}}}\}_{a,x}$ is no-signalling iff it admits a common dilation.

No-signalling

$$\sum_a \widetilde{\widetilde{S_{a|x}}} = \widetilde{\widetilde{S}}$$

$\forall\, x$

$\Longleftrightarrow$

Common dilation

$\exists\, \widetilde{\widetilde{S}}, \{A_{a|x}\}_a$ such that

$$\widetilde{\widetilde{S_{a|x}}}(\widetilde{C}) = \mathrm{Tr}_A\Big[(A_{a|x} \otimes \mathbb{1})\widetilde{\widetilde{S}}(\widetilde{C})\Big]$$

Chiribella, Toigo, Umanità, OSID **20**, 1350003 (2013);
Baroni et al., arXiv:2601.05158 (2026)

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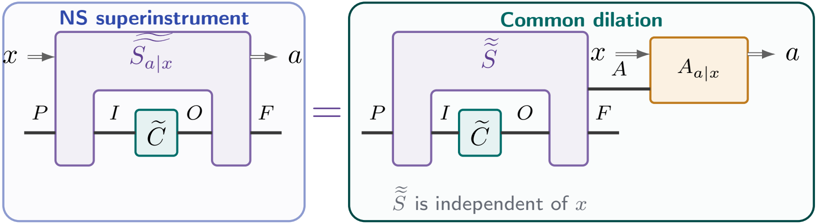
$\{\widetilde{\widetilde{S_{a|x}}}\}_{a,x}$ is no-signalling iff it admits a common dilation.



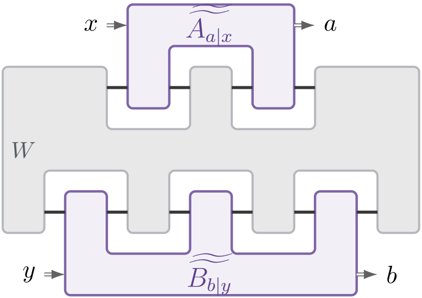
Chiribella, Toigo, Umanità, OSID **20**, 1350003 (2013);
Baroni et al., arXiv:2601.05158 (2026)

Arbitrary NS compositions are spatial

Theorem

Composition of k no-signalling quantum assemblages produces a k-partite Spatial (Bell) quantum correlation.

Two NS superinstruments in a multi-round network



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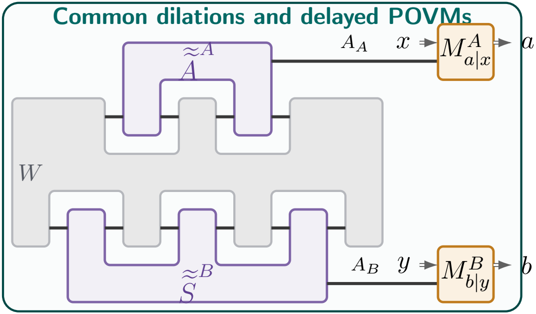


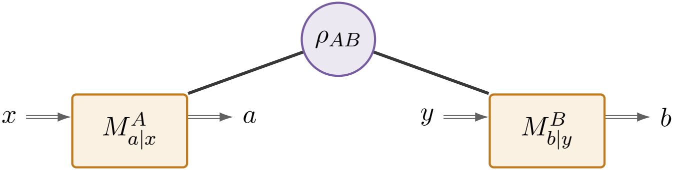
Baroni et al., arXiv:2601.05158 (2026)

Arbitrary NS compositions are spatial

Theorem

Composition of k no-signalling quantum assemblages produces a k-partite Spatial (Bell) quantum correlation.

The network and fixed cores form a shared state; only the delayed POVMs depend on x and y.



Spatial realisation

$$p(a,b|x,y) = \mathrm{Tr}\Big[\rho_{AB}\left(M^A_{a|x} \otimes M^B_{b|y}\right)\Big]$$

Baroni et al., arXiv:2601.05158 (2026)

Unsteerable compositions are Bell-local

Theorem

k unsteerable quantum assemblages	$\Longrightarrow$	**k-partite spatial** Bell-local correlation

Unsteerable compositions are Bell-local

Theorem

k **unsteerable**
quantum assemblages

$\Longrightarrow$

k-**partite spatial**
Bell-local correlation

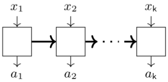
Classical factorisation

$$p(\boldsymbol{a}|\boldsymbol{x}) = \sum_{\boldsymbol{\lambda}} p(\boldsymbol{\lambda}) \prod_{i=1}^{k} p(a_i|x_i, \lambda_i)$$

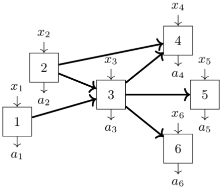
$$\boldsymbol{a} = (a_1, \ldots, a_k),\ \boldsymbol{x} = (x_1, \ldots, x_k),\ \boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_k)$$

The quantum composition induces only shared randomness $p(\boldsymbol{\lambda})$; each setting x_i is processed locally.

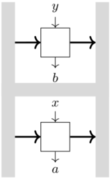
Scenarios covered by our methods



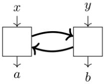
(a) k-sequential.



(b) Directed acyclic graph.



(c) Indefinite causal order.



(d) The simplest loop.

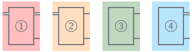
SGHJW beyond states

One simultaneous-purification theorem for every quantum process type

SGHJW beyond states

One simultaneous-purification theorem for every quantum process type

(a)

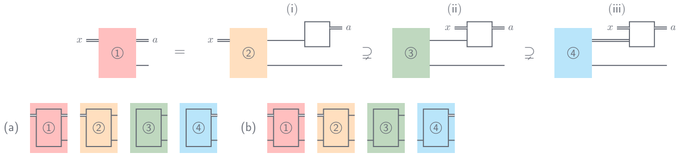


(a) states

(i) arbitrary assemblage (ii) non-signalling (iii) unsteerable

SGHJW beyond states

One simultaneous-purification theorem for every quantum process type



(a) states **(b)** instruments

(i) arbitrary assemblage (ii) non-signalling (iii) unsteerable

SGHJW beyond states

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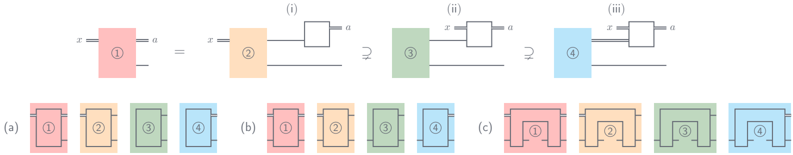

(a) states **(b)** instruments **(c)** superinstruments

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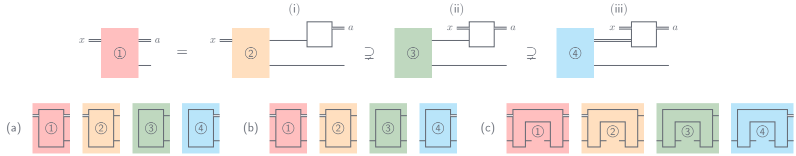


(a) states **(b)** instruments **(c)** superinstruments

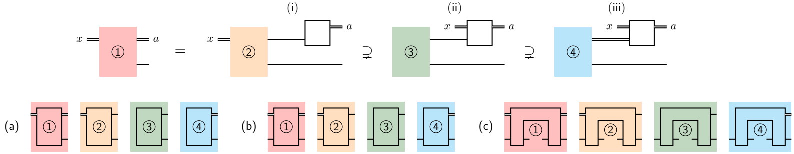
(i) arbitrary assemblage (ii) non-signalling (iii) unsteerable

For *any* quantum object O—state, channel, instrument, comb, process matrix, or higher-order map—

$$\sum_a O_{a|x} = O \quad \forall x \quad \Longleftrightarrow \quad O_{a|x} = \mathrm{Tr}_A\left[(A_{a|x} \otimes \mathbb{1})\,\Omega_{AO}\right],$$

where Ω_{AO} is a single pure dilation of the *same process type* and only the POVM depends on x.

Non-signalling in OPTs and GPTs



(a) states **(b)** instruments **(c)** superinstruments

(i) arbitrary assemblage (ii) non-signalling (iii) unsteerable

$$\sum_a O_{a|x} = O \qquad \forall x$$

Outlook

What we showed

✓ SGHJW extends from states to instruments, combs, and arbitrary quantum objects.

✓ NS is exactly what makes temporal correlations spatial.

✓ Generalises the bipartite result of Wright, Farkas, *Phys. Rev. Lett.* **131**, 220202 (2023) to multipartite scenarios.

✓ Simple and practical.

Consequence: leaving the spatial quantum set requires **internal signalling somewhere**.

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What we showed

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✓ Simple and practical.

Next?

▷ A robust theorem for *approximately* non-signalling assemblages.

▷ Quantify how much signalling is needed to leave the spatial quantum set—and where it must occur.

▷ Infinite-dimensional case?

▷ Composable purification in GPTs and OPTs.

Consequence: leaving the spatial quantum set requires **internal signalling somewhere**.

The team behind this work

Thank you to my co-authors!

Matilde Baroni

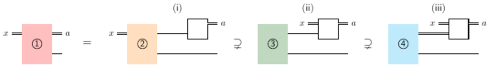
Dominik Leichtle

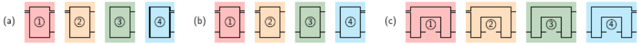
Ivan Šupić

Damian Markham

Thank you!

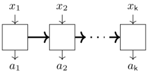
One simultaneous-purification theorem for every quantum process type



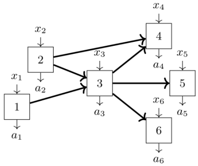


(a) states **(b)** instruments **(c)** super-instruments

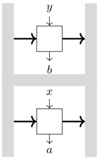
(i) arbitrary assemblage (ii) non-signalling: delayed POVM (iii) unsteerable: classical memory



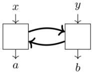
(a) k-sequential.



(b) Directed acyclic graph.



(c) Indefinite causal order.



(d) The simplest loop.

Local non-signalling	$\Longrightarrow$	**spatial correlations**
Local unsteerability	$\Longrightarrow$	**classical spatial correlations**

Baroni et al., arXiv:2601.05158 (2026)