Understanding the relationship between measurement incompatibility, EPR steering, and Bell nonlocality

Marco Túlio Quintino

Sorbonne University

90 years of EPR, Bad Honnef, 08/04/2026

Quantum state

$$\rho \geq 0, \ \mathrm{tr}(\rho) = 1$$

Quantum measurement entanglement

$$M_i \geq 0, \quad \sum_i M_i = \mathbb{1}$$

State+Measurement

$$p(i|\rho, \{M_i\}_i) = \mathrm{tr}(\rho M_i)$$

State+Measurement

$$p(i|\rho, \{M_i\}_i) = \mathrm{tr}(\rho M_i), \quad |\langle i|\psi\rangle|^2$$

Quantum entanglement

$$\rho_{AB} \neq \sum_{\lambda} \pi(\lambda) \rho_A^{\lambda} \otimes \rho_B^{\lambda}$$

A brief digression

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- 1964: The singlet is Bell nonlocal
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- 2006: Wiseman, Jones, Doherty formalised EPR steering for mixed states
- Maybe entanglement should be EPR-stering? Or, Bell NL (LHV)? Well. . .

Quantum entanglement

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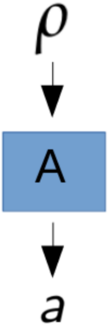
How to extract strong correlations from quantum states?

Quantum measurement

$$p(a|\rho, A) = \mathrm{tr}(\rho A_a)$$

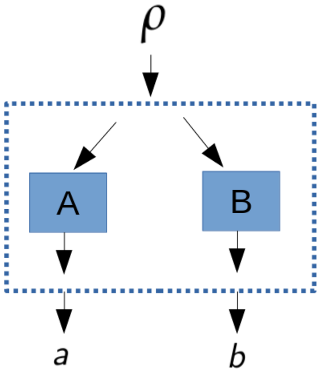
$$A = \{A_a\}, \quad A_a \geq 0, \quad \sum_a A_a = I$$



Joint Measurability

$\{A_a\}$ and $\{B_b\}$ are JM if there exists a measurement $\{M_{ab}\}$ s. t.:

$$\sum_a M_{ab} = B_b$$
$$\sum_b M_{ab} = A_a$$



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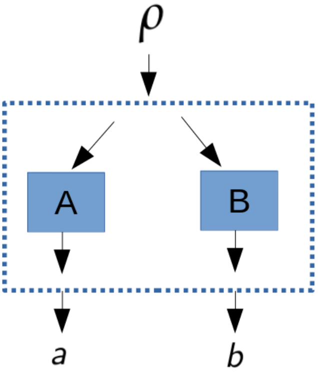

Commutation and joint measurability

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- Also, $\sum_a A_a B_b = B_b$ and $\sum_b A_a B_b = A_a$
- But, JM does not imply commutation

Pauli Measurements

$$\sigma_Z : \{|0\rangle\langle 0|\,,\ |1\rangle\langle 1|\} \quad \sigma_X : \{|+\rangle\langle +|\,,\ |-\rangle\langle -|\}$$

Noise Pauli Measurements

$$\sigma_{Z,\eta} : \left\{ \eta \left|0\right\rangle\!\left\langle 0\right| + (1-\eta)\frac{I}{2} \; ; \qquad \eta \left|1\right\rangle\!\left\langle 1\right| + (1-\eta)\frac{I}{2} \right\}$$

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$$\eta \leq \frac{1}{\sqrt{2}} \implies \text{Joint Measurability}$$

Hollow Triangle

$$\sigma_{Z,\eta} : \left\{ \eta \, |0\rangle\langle 0| + (1-\eta)\frac{I}{2} \; ; \qquad \eta \, |1\rangle\langle 1| + (1-\eta)\frac{I}{2} \right\}$$

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$$\sigma_{Y,\eta} : \left\{ \eta \, |Y+\rangle\langle Y+| + (1-\eta)\frac{I}{2} \; ; \qquad \eta \, |Y-\rangle\langle Y-| + (1-\eta)\frac{I}{2} \right\}$$

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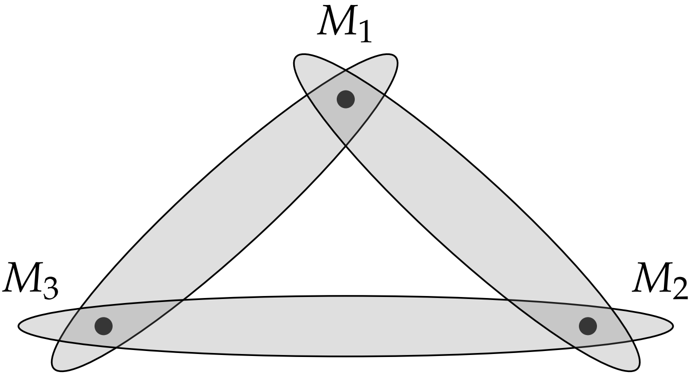
$$\eta \leq \frac{1}{\sqrt{2}} \implies \text{Pairwise Measurability}$$

$$\eta \leq \frac{1}{\sqrt{3}} \implies \text{Triplewise Measurability}$$

Hollow Triangle



T. Heinosaari, D. Reitzner, P. Stano: Foundations of Physics (2008)

Joint Measurability

A set of measurements $A_{a|x}$ is JM if there exists a single measurement $\{M_\lambda\}$ and a classical post-processing $p(a|x, \lambda)$ s. t.:

$$M_{a|x} = \sum_\lambda p(a|x, \lambda) E_\lambda$$

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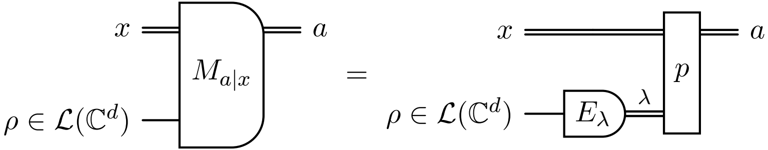
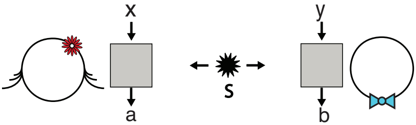


Measurement compatibility, EPR steering, and Bell nonlocality

Measurement compatibility and quantum correlations?

Bell Nonlocality



The Correlation/Anticorrelation Game

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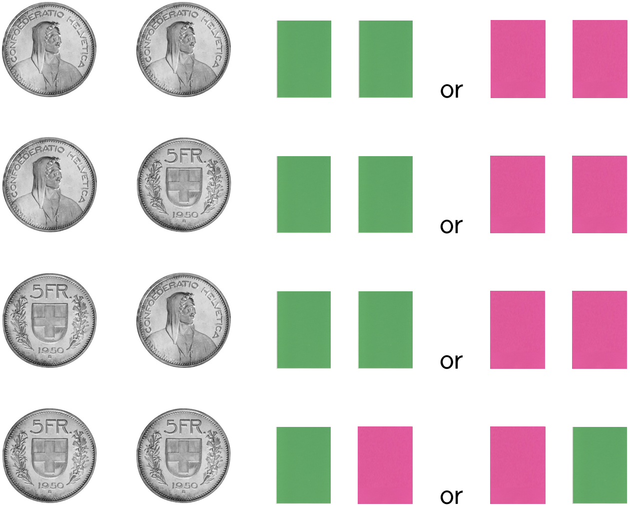
Winning Conditions



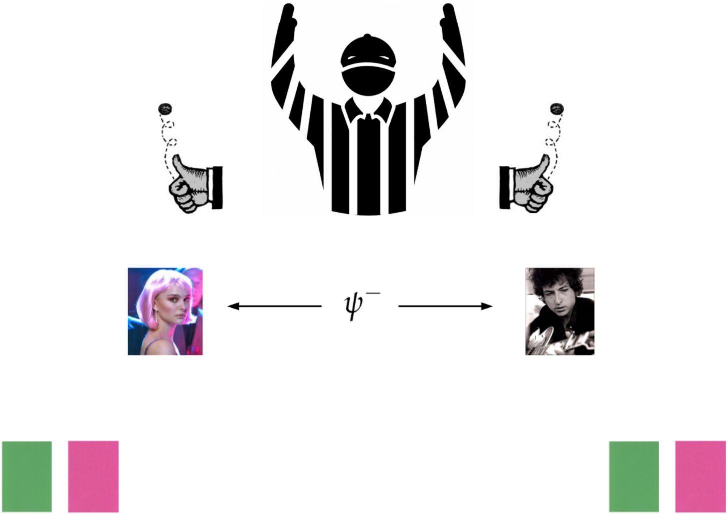
Best Strategy

Can Alice and Bob always win?

Best Strategy

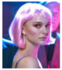
Best (classical) strategy wins with probability $\frac{3}{4}$

Quantum Strategy



Quantum Strategy

$$p_q = \frac{2+\sqrt{2}}{4} \approx 0.8535$$



$$\longleftarrow \quad \psi^- \quad \longrightarrow$$

Quantum Strategy

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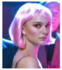
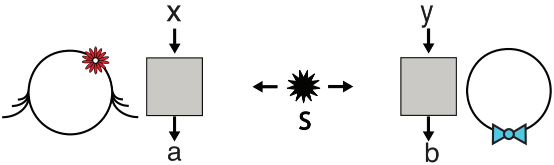
 $\longleftarrow \quad \psi^- \quad \longrightarrow$ 

But, what if Alice's measurements are JM?

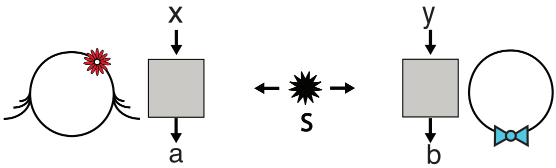
Bell nonlocality

$$p_{\text{win}} = \sum_{abxy} \pi(x,y) V(ab|xy) p(ab|xy)$$



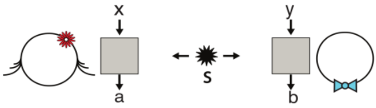
Bell nonlocality

$$p(ab|xy) \neq \sum_{\lambda} \pi(\lambda) p_A(a|x,\lambda) p_B(b|y,\lambda)$$

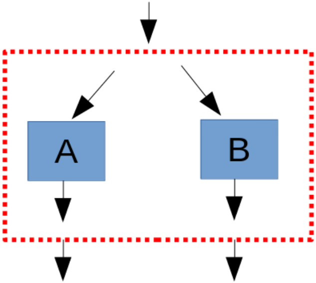


Bell NL and no JM

Bell NL



No JM



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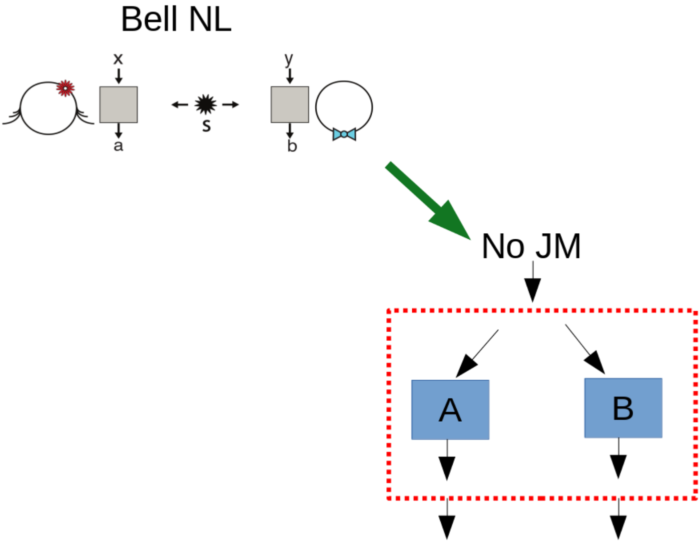
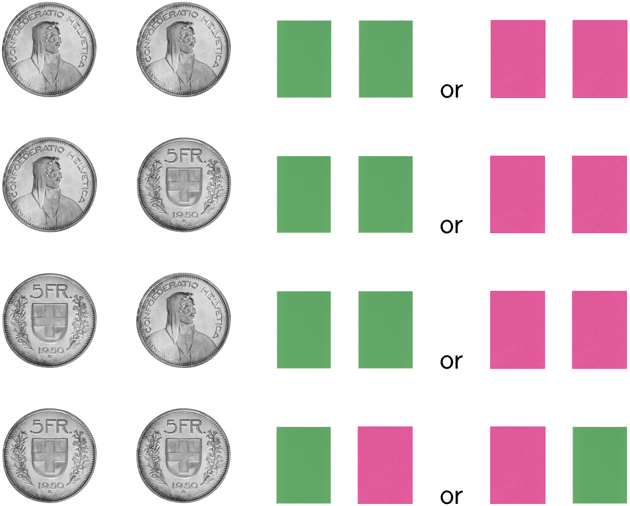


Bell NL

Correlator:

$$\langle A_x B_y \rangle = p(a = b|xy) - p(a \neq b|xy)$$

Winning Conditions



Bell NL

Correlator:

$$\langle CHSH \rangle := \langle A_1 B_1 \rangle + \langle A_1 B_2 \rangle + \langle A_2 B_1 \rangle - \langle A_2 B_2 \rangle$$

Bell NL

The CHSH inequality:

$$\langle CHSH \rangle := \langle A_1 B_1 \rangle + \langle A_1 B_2 \rangle + \langle A_2 B_1 \rangle - \langle A_2 B_2 \rangle \overset{\text{local}}{\leq} 2$$

Bell NL

Quantum Observable:

$$A_x := A_{g|x} - A_{p|x}$$

$$B_x := B_{g|x} - B_{p|x}$$

A_x and B_y are matrices with eigenvalues ± 1

$$\langle A_x B_y \rangle_Q = \mathrm{tr}(\rho A_x \otimes B_y)$$

Bell NL

The CHSH operator:

$$CHSH := A_1 \otimes B_1 + A_1 \otimes B_2 + A_2 \otimes B_1 - A_2 \otimes B_2$$

Bell NL

The CHSH operator:

$$CHSH := A_1 \otimes B_1 + A_1 \otimes B_2 + A_2 \otimes B_1 - A_2 \otimes B_2$$

Maximal quantum score is given by the largest eigenvalue of CHSH

Tsirelson Bound 2.0

SYMPOSIUM ON THE FOUNDATIONS OF MODERN PHYSICS, pp. 441-460
edited by P. Lahti & P. Mittelstaedt

QUANTUM AND QUASI-CLASSICAL ANALOGS OF BELL INEQUALITIES*

L.A.Khalfin, B.S.Tsirelson

Steklov Mathematical Institute, Leningrad D-11, USSR

Both (1) and (2) can be treated as a consequences of the following inequality:

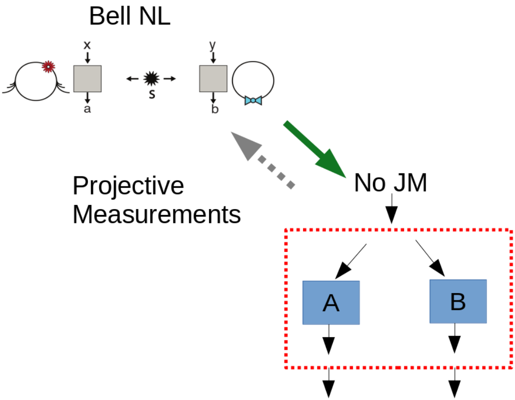
$$(A_1B_1 + A_1B_2 + A_2B_1 - A_2B_2)^2 \leqslant 4\cdot\mathbb{1} - [A_1, A_2]\cdot[B_1, B_2], \qquad (3)$$

holding under the same assumptions as (2). From (3) it follows immediately

$$\|A_1B_1 + A_1B_2 + A_2B_1 - A_2B_2\| \leqslant \sqrt{4 + \|[A_1, A_2]\|\cdot\|[B_1, B_2]\|}. \qquad (4)$$

$$\|CHSH\| = \sqrt{4I + \|[A_1, A_2]\|\,\|[B_1, B_2]\|}$$

Bell NL and JM

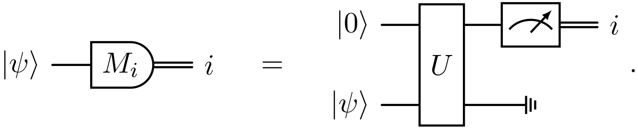


Bell NL and JM

Beyond projective measurements?
Beyond CHSH?

Naimark dilation measurement

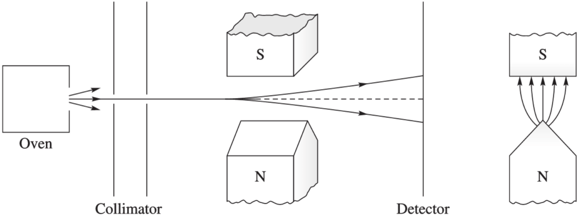
For every POVM $\{M_i\}$, there exists a unitary U such that:



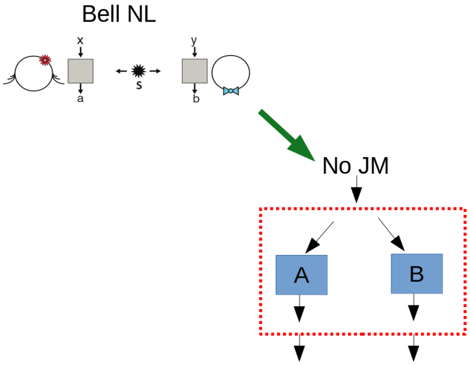
Quantum measurement

Naimark dilation:

POVM $\iff$ global unitary + measurement



Bell NL and JM



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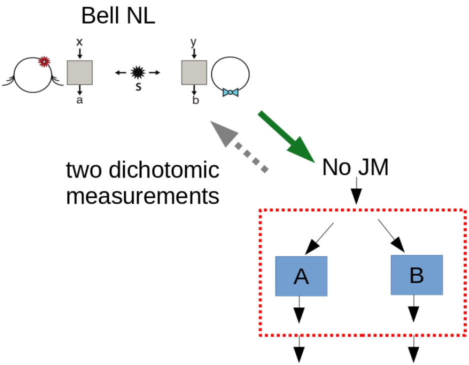
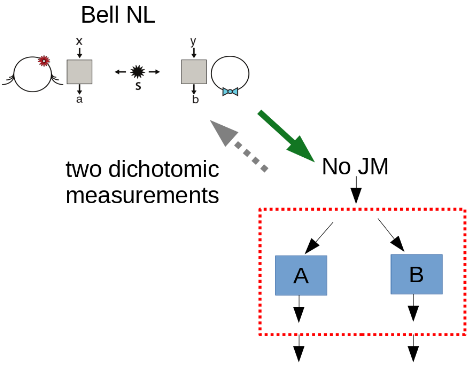


M. Wolf, D. Perez-Garcia, C. Fernandez, PRL (2009)

Bell NL and JM

$\{A_{a|x}\}_{a,x=1}^{2}$ not JM $\implies$ $\exists \rho_{AB}$ and $\{B_{b|y}\}$ such that:
$p(ab|xy) = \mathrm{tr}\big(\rho_{AB}\, A_{a|x} \otimes B_{b|y}\big)$ is Bell NL

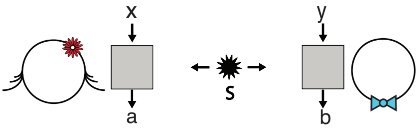


M. Wolf, D. Perez-Garcia, C. Fernandez, PRL (2009)

EPR Steering

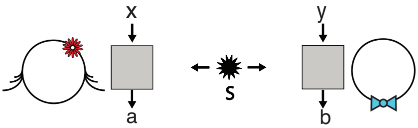
Another manifestation of quantum non-locality

Bell nonlocality



$$p(ab|xy) \neq \sum_{\lambda} \pi(\lambda) p_A(a|x,\lambda) p_B(b|y,\lambda)$$

Bell nonlocality and EPR Steering



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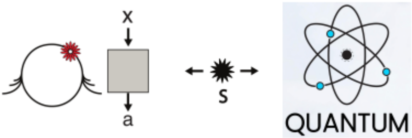

$$p(ab|xy) \neq \sum_{\lambda} \pi(\lambda) p_A(a|x,\lambda) \operatorname{tr}\left(\rho_\lambda B_{b|y}\right)$$

EPR steering

- If Bob devices are characterised, he can perform tomography and obtain the *assemblage*

$$\sigma_{a|x} = \mathrm{tr}_A(\rho_{AB} A_{a|x} \otimes I)$$

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- Steering as semi-device indepdendent entanglement certification

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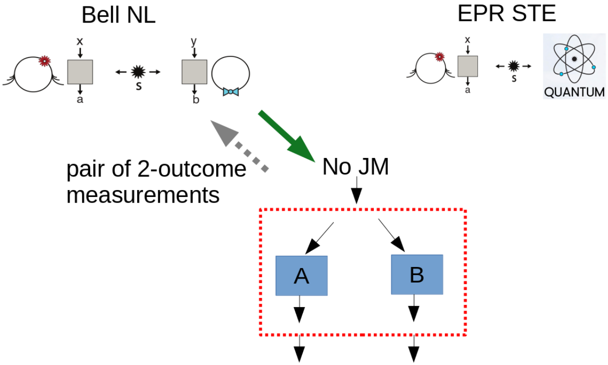
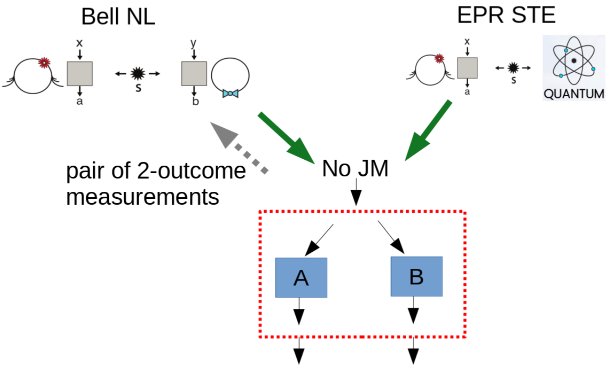
- Steering as semi-device indepdendent entanglement certification
- If $\sigma_{a|x} = \sum_\lambda \pi(\lambda) p_A(a|x,\lambda)\rho_\lambda^B$, then

$$\begin{aligned} p(ab|xy) &= \mathrm{tr}\left(\sigma_{a|x} B_{b|y}\right) \\ &= \sum_\lambda \pi(\lambda) p_A(a|x,\lambda)\, \mathrm{tr}\left(\rho_\lambda^B B_{b|y}\right) \end{aligned}$$

Bell NL, JM, and EPR STE



Bell NL, JM, and EPR STE



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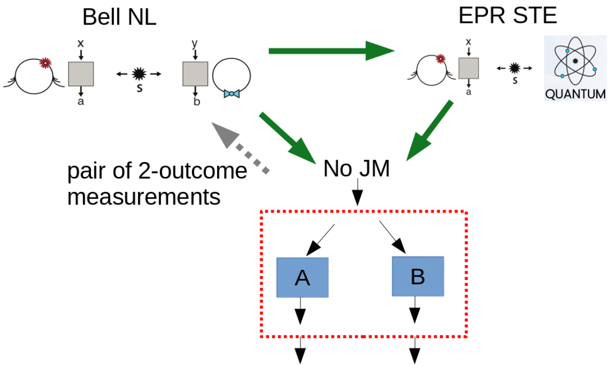

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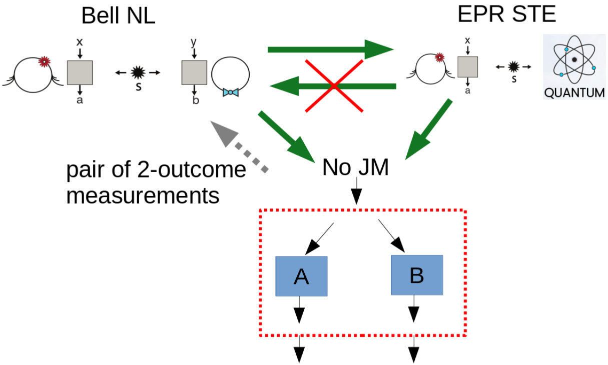
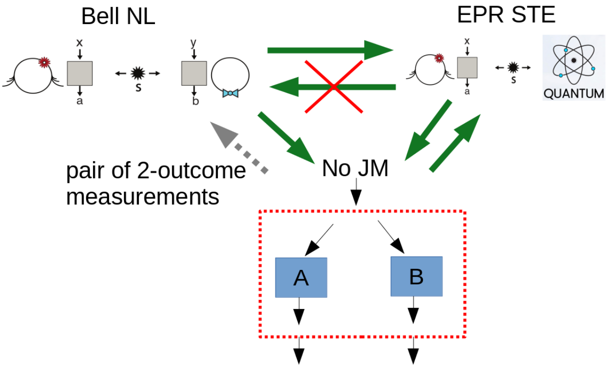


H. M. Wiseman, S. J. Jones, A. C. Doherty, PRL (2007)
A. Acin, N. Gisin, and .B Toner, PRA (2006)
B. S. Tsirelson, J. Soviet Math. (1987)
M.T. Quintino, T. Vertesi, N. Brunner, D. Cavalcanti, R. Augusiak, M. Demianowicz, A. Acín, N. Brunner PRA (2015)

Bell NL, JM, and EPR STE

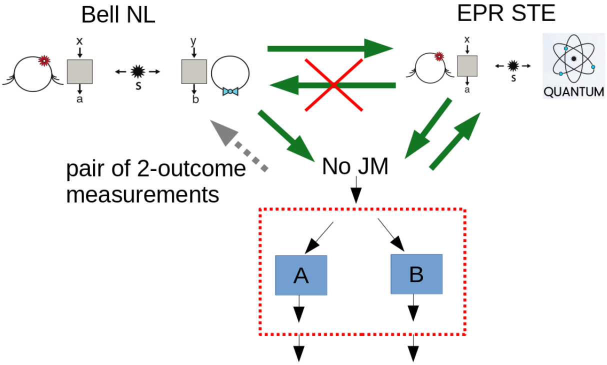


M.T. Quintino, T. Vertesi, N. Brunner PRL (2014)
R. Uola, T. Moroder, and O. Gühne PRL (2014)

Bell NL, JM, and EPR STE

$\{A_{a|x}\}$ not JM
ρ_{AB} is pure and entangled (full Schmidt rank)
$\{B_{b|y}\}$ is informationally complete:

$$p(ab|xy) = \mathrm{tr}\left(\rho_{AB}\, A_{a|x} \otimes B_{b|y}\right) \text{ is EPR Steerable}$$



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no-JM $\cong$ EPR STE:

- For the max ent state, we have:

$$\sigma_{a|x} = \mathrm{tr}_A(\left|\phi_d^+\middle\rangle\middle\langle\phi_d^+\right| A_{a|x} \otimes \mathbb{1}) = \frac{A_{a|x}^T}{d}$$

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- Local filtering equivalent classes:

$$\mathbb{1} \otimes F_B |\phi_d^+\rangle = |\psi\rangle_{AB}$$

no-JM $\cong$ EPR STE:

- For the max ent state, we have:

$$\sigma_{a|x} = \mathrm{tr}_A(\left|\phi_d^+\right\rangle\!\left\langle\phi_d^+\right| A_{a|x} \otimes \mathbb{1}) = \frac{A_{a|x}^T}{d}$$

- Local filtering equivalent classes:

$$\mathbb{1} \otimes F_B \left|\phi_d^+\right\rangle = |\psi\rangle_{AB}$$

- Local filtering (on Bob's side) cannot create steering.

Consequences:

- Models for isotropic state = critical visibility for performing all measuerements: $\max_\eta$ such that $A^{\eta}_{a|x} = \eta A_{a|x} + (1-\eta)\frac{\mathbb{1}}{d}\operatorname{tr}\big(A_{a|x}\big)$ is JM.

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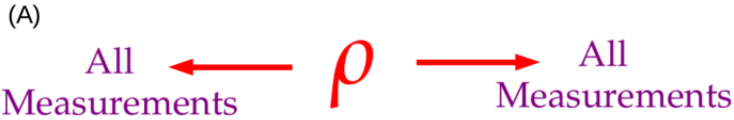
  Y. Zhang, E. Chitambar PRL (2024), Martin J. Renner PRL (2024)
- Incompatibility breaking and steering breaking channels are the same

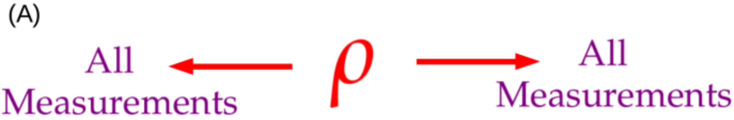
  T. Heinosaari, J. Kiukas, D. Reitzner, J. Schultz, JPA (2015)

How about JM and Bell nonlocality?

Local hidden variable model



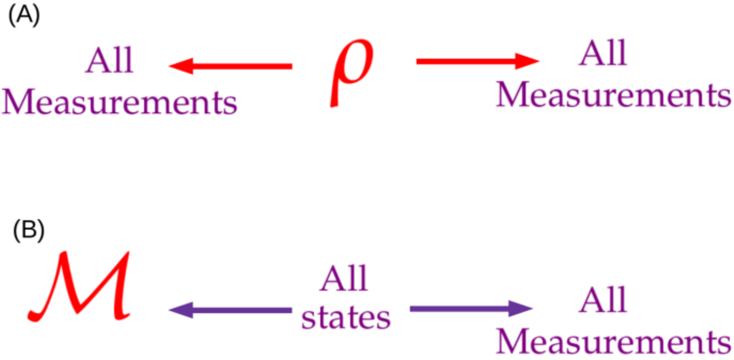
Local hidden variable model



$$W_2 := \eta \left|\phi_2^+\right\rangle\!\left\langle\phi_2^+\right| + (1-\eta)\frac{I}{4}$$

Entangled states addmiting LHV models
R. Werner, PRA (1989)

Local hidden variable model



Bell nonlocality

$$p(ab|xy) = \mathrm{tr}\big(\rho^{\eta}_{AB}\, A_{a|x} \otimes B_{b|y}\big)$$

$$\rho^{\eta}_{AB} := \eta\rho_{AB} + (1-\eta)\frac{I}{d} \otimes \rho_B$$

$$\rho_B := \mathrm{tr}_A(\rho_{AB})$$

Bell nonlocality

$$\mathrm{tr}\big(\rho^{\eta}_{AB}\, A_{a|x} \otimes B_{b|y}\big) = \mathrm{tr}\Big(\rho_{AB}\, A^{\eta}_{a|x} \otimes B_{b|y}\Big)$$

$$A^{\eta}_{a|x} := \eta A_{a|x} + (1-\eta)\frac{I}{d}\,\mathrm{tr}\big(A_{a|x}\big)$$

Local hidden variable model

LHV model for the class:

$$W_{\eta,\theta} := \eta \left|\phi^\theta\middle\rangle\!\middle\langle\phi^\theta\right| + (1-\eta)\frac{I}{2} \otimes \rho_\theta$$

$$\left|\phi^\theta\right\rangle := \sin(\theta)\left|00\right\rangle + \cos(\theta)\left|11\right\rangle , \quad \rho_\theta := \mathrm{tr}_A(\left|\phi^\theta\middle\rangle\!\middle\langle\phi^\theta\right|$$

in a range where there are noisy incompatible measurements

Local hidden variable model

$$W_{\eta,\theta} := \eta \left|\phi^{\theta}\middle\rangle\middle\langle\phi^{\theta}\right| + (1-\eta)\frac{I}{2} \otimes \rho_{\theta}$$

$$\cos^2(\theta) \geq \frac{2\eta - 1}{(2-\eta)\eta^3} \implies \text{LHS model}$$

J. Bowles, F. Hirsch, M.T. Quintino, N. Brunner, PRL (2016)

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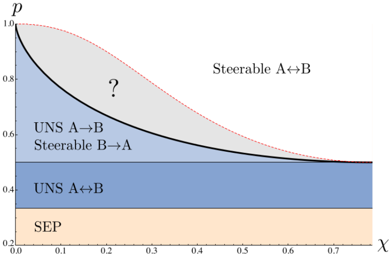

J. Bowles, F. Hirsch, M.T. Quintino, N. Brunner, PRL (2016)

Local hidden variable model

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For $\eta > 1/2$, $\{A_{a|x}\}$ is not JM.
But for $\theta = \pi/4$ the model returns $\eta = 1/2 \ldots$

Local hidden variable model

For the maximally entangled state, we have Grothendieck!

Local hidden variable model

Journal of Soviet Mathematics,
vol. 36, number 4, pp. 557–570.
February, 1987

QUANTUM ANALOGUES OF THE BELL INEQUALITIES. THE CASE OF TWO SPATIALLY SEPARATED DOMAINS

B. S. Tsirel'son

UDC 519.2

significantly simpler manner in terms of vectors in a Euclidean space. Then it becomes clear that the constant $K_\cdot$ is nothing else but Grothendieck's well known constant K_G, investgated by mathematicians from 1956 up to now!

Regarding the problem of the Grothendieck constant, we refer the reader to [11]. It is proved there that

$$K_G \leqslant \frac{\pi}{2\ln(1+\sqrt{2})} \approx 1{,}782.$$

Local hidden variable model

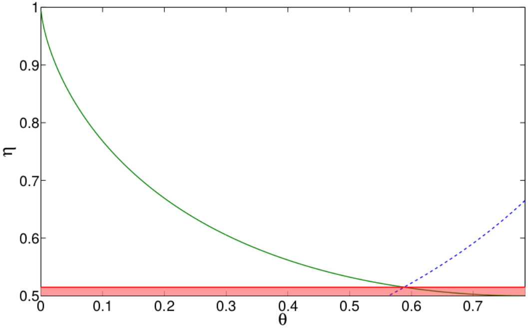
$$W_\eta := \eta \left|\phi^+\right\rangle\!\left\langle\phi^+\right| + (1-\eta)\frac{I}{4}$$

LHV for $\eta \leq \frac{1}{K_G(3)}$, and $K_G(3) < 2$

A. Acin, N. Gisin, B. Toner, PRA (2006)
B. Tsirelson, J. Soviet Math. (1987)
J. L. Krivine, Adv. Math. (1979)

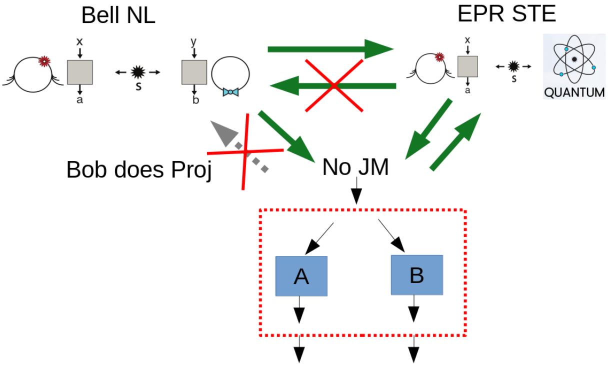
Bell NL and JM

LHV Model+Grothendieck+LHV extention based on PPT:



M.T. Quintino, J. Bowles, F. Hirsch, N. Brunner, PRA (2016)

Bell NL, JM, and EPR STE



Bell NL, JM, and EPR STE

Nice, but…

Bob can do POVMs…

Bell NL, JM, and EPR STE

Nice, but...

Bob can do POVMs...

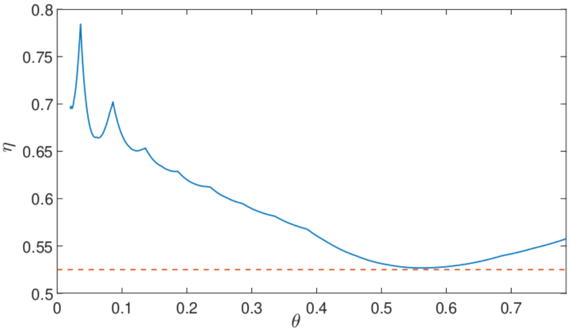
Normally, constructing LHV models is requires a lot of creativity

Local hidden variable model

Let the computer find LHV models for us!

F. Hirsch, M.T. Quintino, T. Vertesi, M. Pusey, N. Brunner, PRL (2016)
D. Cavalcanti, L. Guerini, R. Rabelo, P. Skrzypczyk, PRL (2016)

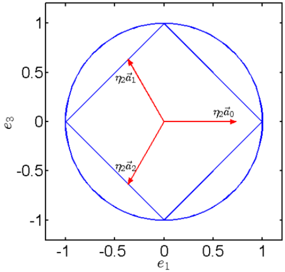
Bell NL and JM

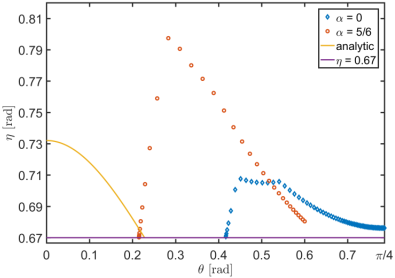


F. Hirsch, M.T. Quintino, N. Brunner, PRA (2018)

Bell NL and JM

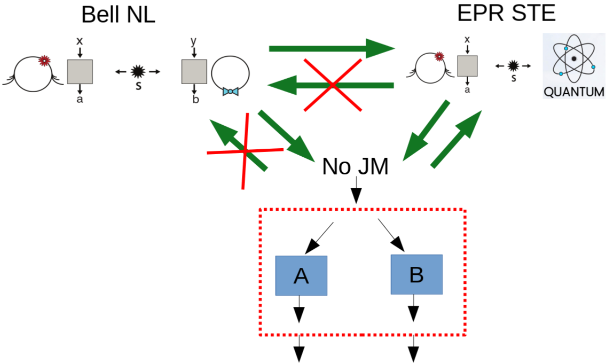
Three planar measurements:





E. Bene, T. Vertesi, NJP (2018)

Bell NL, JM, and EPR STE



Bell NL and JM

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- How about multipartite scenarios?

Bell NL and JM

- $\{M_{a|x}\}$ is not JM, but useless for Bell NL.
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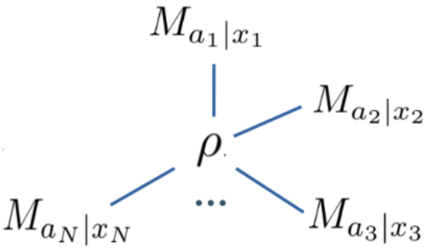
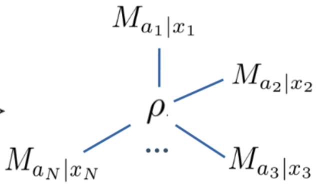
- How about multipartite scenarios?
- How about an N-partite Bell scenario where All parties perform the same measurement



Multipartite Bell NL and JM

$$\{M_{a|x}\} \text{ is not JM } \Longrightarrow$$



is not Bell local

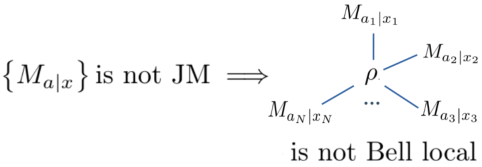
All incompatible measurements on qubits lead to multiparticle Bell nonlocality
M. Plávala, O. Gühne, M.T. Quintino Phys. Rev. Lett. 134, 200201 (2025)

Multipartite Bell NL and JM

If $d = 2$ and $\{M_{a|x}\}$ is not JM, there exists a number of parties N and a state ρ such that

$$p(a_1 \dots a_N | x_1 \dots x_N) = \mathrm{tr}\big(\rho \, M_{a_1|x_1} \otimes \dots \otimes M_{a_N|x_N}\big)$$

is Bell NL



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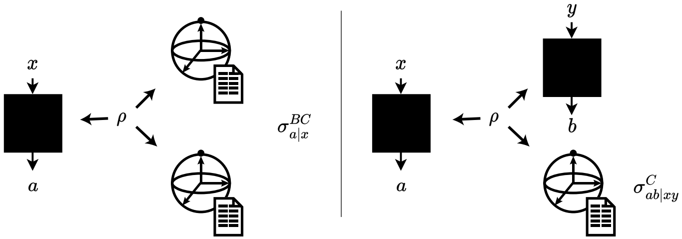
Multipartite Bell NL and JM

If there are incompatible measurements that cannot lead to Bell NL in the multipartite scenario, there exists an NPT bound entangled state

All incompatible measurements on qubits lead to multiparticle Bell nonlocality
M. Plávala, O. Gühne, M.T. Quintino Phys. Rev. Lett. 134, 200201 (2025)

Multipartite steering?

$$\rho^{ABC}_{\text{Bi-Sep}} = \sum_\lambda p(\lambda)\rho^A_\lambda \otimes \rho^{BC}_\lambda \; + \sum_\mu p(\mu)\rho^B_\mu \otimes \rho^{AC}_\mu + \sum_\nu p(\nu)\rho^C_\nu \otimes \rho^{AB}_\nu$$



Multipartite steering?

- THM[1]: If $\{M_{a|x}\} \notin$JM, $\forall N > 1$, $\exists \rho_N$ s.t.

$$\sigma_{a|x} = \mathrm{tr}_A(M_{a|x} \otimes \mathbb{1}_2 \otimes \ldots \otimes \mathbb{1}_N \; \rho_N)$$

exhibits genuine multipartite steering

[1] Can every set of incompatible measurements lead to genuine multipartite steering? L.E.A. Porto, L. Tendick, D. Cavalcanti, R. Uola, M.T. Quintino, arXiv (2026)

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- Just set $\rho = |\psi\rangle\langle\psi|$ with

$$|\psi\rangle := \sum_{i=0}^{d-1} \sqrt{p_i}\, |i\rangle \otimes |\phi_i\rangle_{2,\ldots,N}$$

  $|\phi_i\rangle$ are orthogonal and symmetric under permutations of the $N-1$ parties.

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- New concepts? Proof techniques? Poster of Lucas Porto

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Summary

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- Not all incompatible measurements can lead to Bell NL
- But with many parties, all qubit incompatible measurements can lead to Bell NL (for $d > 2$ a counterexample would prove the existence of NPT bound entangled states)
- Compatible measurements and multipartite unsteerable assemblages are deeply related (if only one party is uncharacterised)
- In general, the relationship between measurement compatibility and mutlipartite steering is less evident

Open questions

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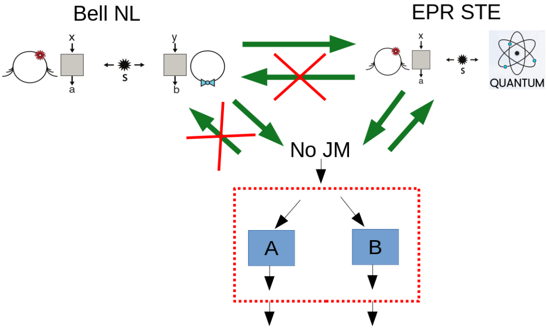
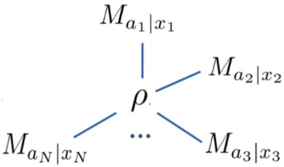
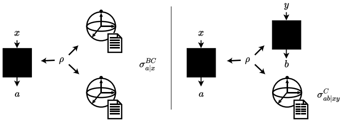
- Natural manners to "activate" measurement NL
  (how about d>2)?
- Multipartite measurement compatibility?
  (See the poster of Lucas Porto)
- Bell Local measurements and other concepts in quantum info?

Thank you!


Bell NL
x
a
s
y
b
EPR STE
x
a
s
QUANTUM
No JM
A
B


$M_{a_1|x_1}$
$M_{a_2|x_2}$
ρ
$M_{a_3|x_3}$
$M_{a_N|x_N}$
...


x
a
ρ
$\sigma^{BC}_{a|x}$
x
a
ρ
y
b
$\sigma^{C}_{ab|xy}$