All incompatible measurements on qubits lead to multiparticle Bell nonlocality

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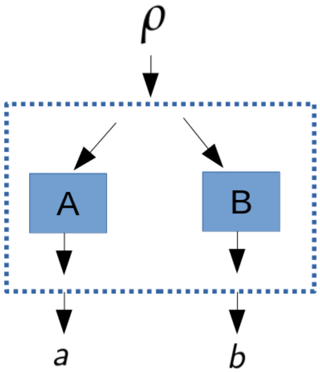
Measurement incompatibility



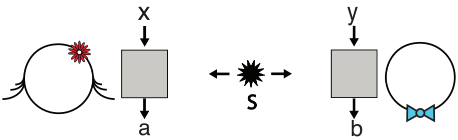
Quantum entanglement

$$\rho_{AB} \neq \int \pi(\lambda)\rho_A^\lambda \otimes \rho_B^\lambda \, \mathrm{d}\lambda$$

State+Measurement

$$p(i|\rho, \{M_i\}_i) = \mathrm{tr}(\rho M_i)$$

Bell nonlocality



$$p(ab|xy) = \mathrm{tr}\big(\rho_{AB} A_{a|x} \otimes B_{b|y}\big)$$

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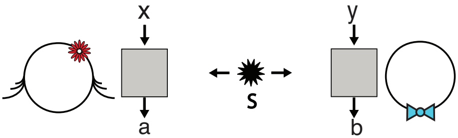
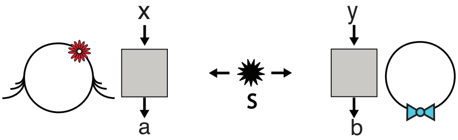


$$p(ab|xy) = \mathrm{tr}\left(\rho_{AB} A_{a|x} \otimes B_{b|y}\right)$$

$$p(ab|xy) \neq \sum_{\lambda} \pi(\lambda) p_A(a|x,\lambda) p_B(b|y,\lambda)$$

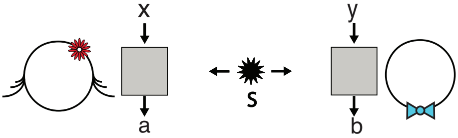
Bell nonlocality



$$p(ab|xy) \neq \sum_{\lambda} \pi(\lambda) p_A(a|x,\lambda) p_B(b|y,\lambda)$$

Bell NL $\implies$ Entanglement + Measurement incompatibility

Bell nonlocality



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Bell NL $\implies$ Entanglement + Measurement incompatibility

Bell NL $\overset{?}{\impliedby}$ Entanglement + Measurement incompatibility

Bell Nonlocality

The Correlation/Anticorrelation Game

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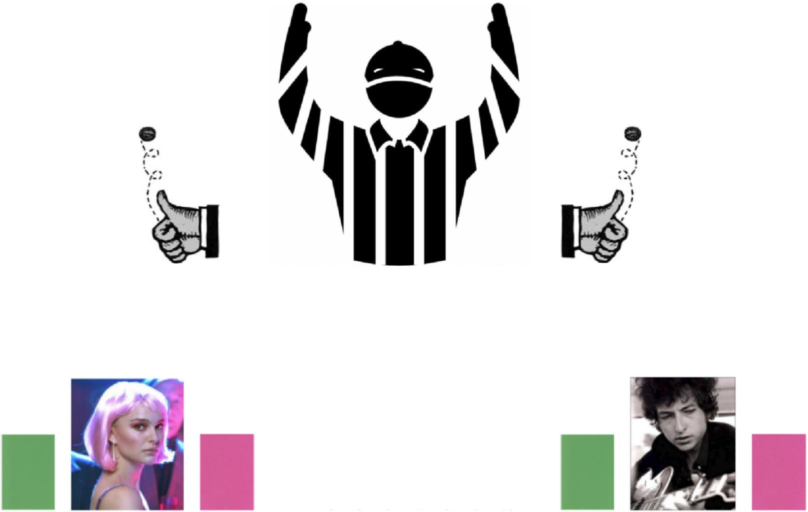
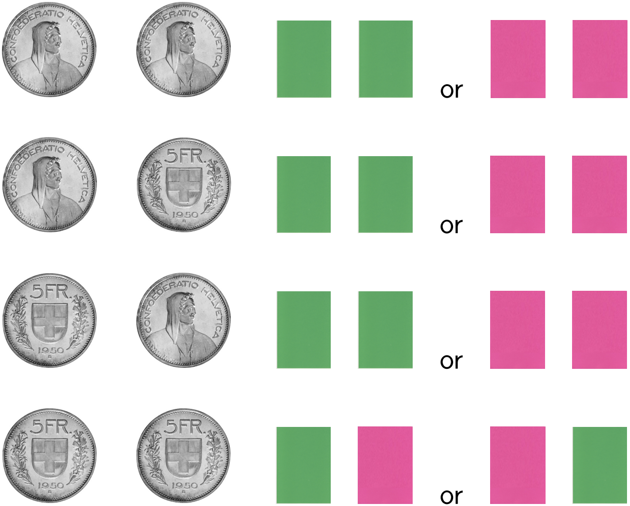


Winning Conditions



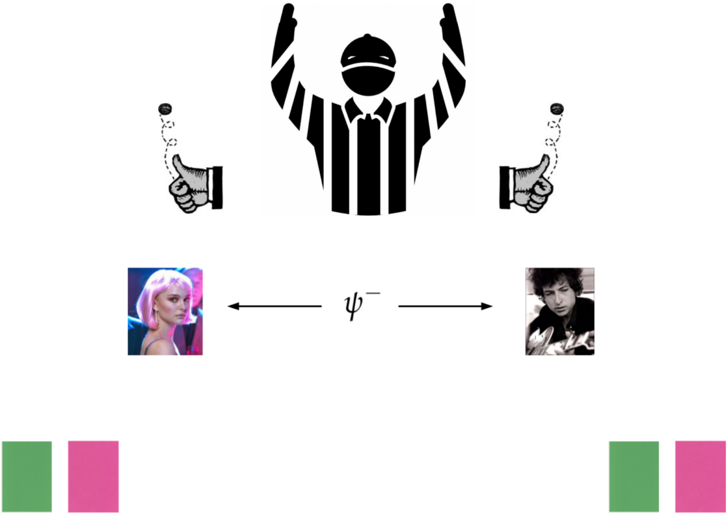
Best Strategy

Can Alice and Bob always win?

Best Strategy

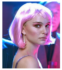
Best (classical) strategy wins with probability $\frac{3}{4}$

Quantum Strategy



Quantum Strategy

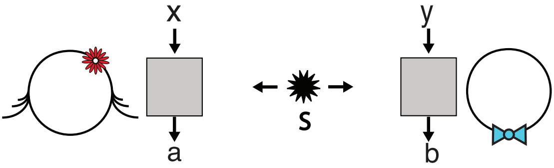
$$p_q = \frac{2+\sqrt{2}}{4} \approx 0.8535$$



$\longleftarrow \quad \psi^- \quad \longrightarrow$

Bell nonlocality

$$p_{\text{win}} = \sum_{abxy} \pi(x,y) V(ab|xy) p(ab|xy)$$



Bell nonlocality

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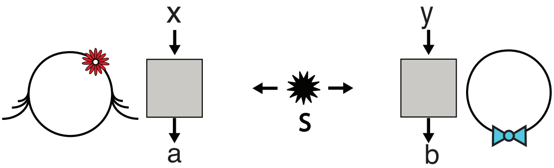
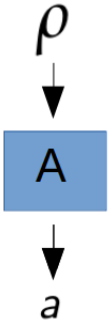


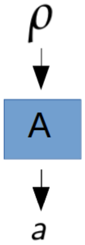
Quantum measurement



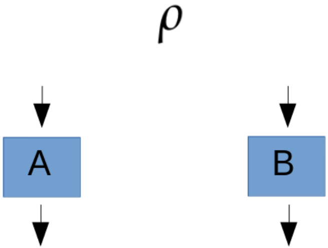
Quantum measurement: POVM

$$p(a|\rho, A) = \mathrm{tr}(\rho A_a)$$

$$A = \{A_a\}, \quad A_a \geq 0, \quad \sum_a A_a = I$$

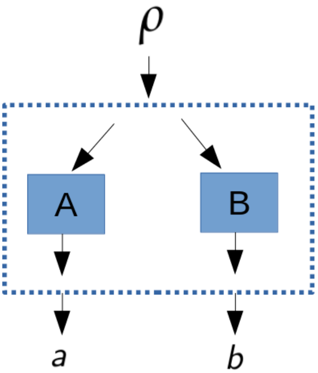


Measurement compatibility



Measurement compatibility

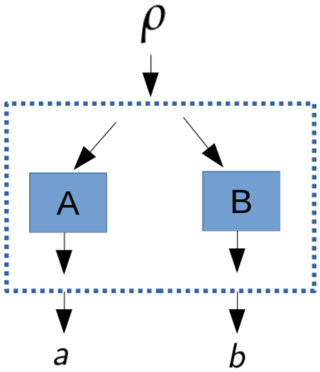
Joint measurability



Joint Measurability

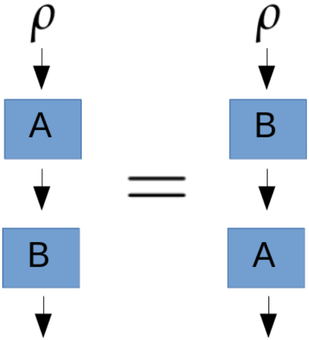
$\{A_a\}$ and $\{B_b\}$ are JM if there exists a measurement $\{M_{ab}\}$ s. t.:

$$\sum_a M_{ab} = B_b$$
$$\sum_b M_{ab} = A_a$$



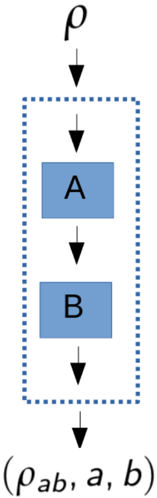
Measurement compatibility

Commuting measurements



Measurement compatibility

Commutation $\implies$ measurement compatibility



Joint Measurability

The set of measurements $A_{a|x}$ is JM if there exists a single measurement $\{M_\lambda\}$ and a classical post-processing $p(a|x,\lambda)$ s. t.:

$$M_{a|x} = \sum_\lambda p(a|x,\lambda) E_\lambda$$

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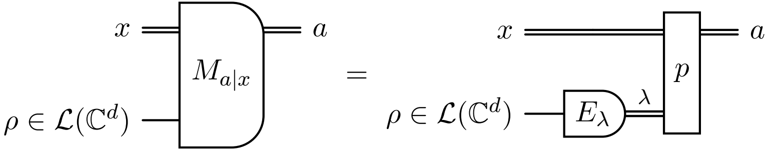

Pauli Measurements

$$\sigma_Z : \{ |0\rangle\langle 0| \, , \; |1\rangle\langle 1| \} \quad \sigma_X : \{ |+\rangle\langle +| \, , \; |-\rangle\langle -| \}$$

Noise Pauli Measurements

$$\sigma_{Z,\eta} : \left\{ \eta \left|0\right\rangle\!\left\langle 0\right| + (1-\eta)\frac{I}{2} \;; \qquad \eta \left|1\right\rangle\!\left\langle 1\right| + (1-\eta)\frac{I}{2} \right\}$$

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$$\eta \leq \frac{1}{\sqrt{2}} \implies \text{Joint Measurability}$$

Hollow Triangle

$$\sigma_{Z,\eta} : \left\{ \eta\, |0\rangle\langle 0| + (1-\eta)\frac{I}{2}\ ; \qquad \eta\, |1\rangle\langle 1| + (1-\eta)\frac{I}{2} \right\}$$

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$$\sigma_{Y,\eta} : \left\{ \eta\, |Y+\rangle\langle Y+| + (1-\eta)\frac{I}{2}\ ; \qquad \eta\, |Y-\rangle\langle Y-| + (1-\eta)\frac{I}{2} \right\}$$

$$\eta \leq \frac{1}{\sqrt{2}} \implies \text{Pairwise Measurability}$$

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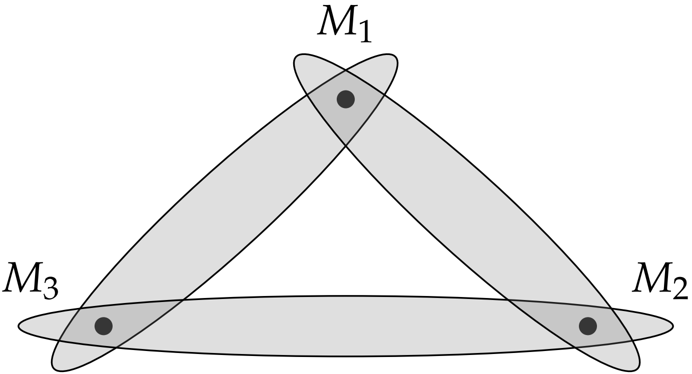
$$\eta \leq \frac{1}{\sqrt{2}} \implies \text{Pairwise Measurability}$$

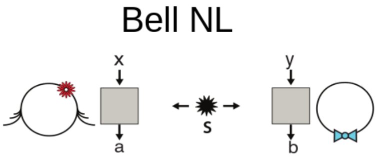
$$\eta \leq \frac{1}{\sqrt{3}} \implies \text{Triplewise Measurability}$$

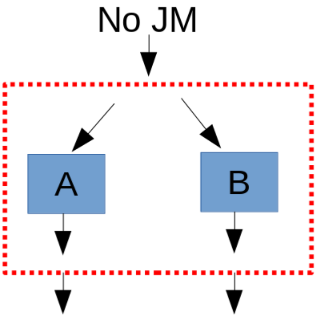
Hollow Triangle



T. Heinosaari, D. Reitzner, P. Stano: Foundations of Physics (2008)

Bell NL and no JM





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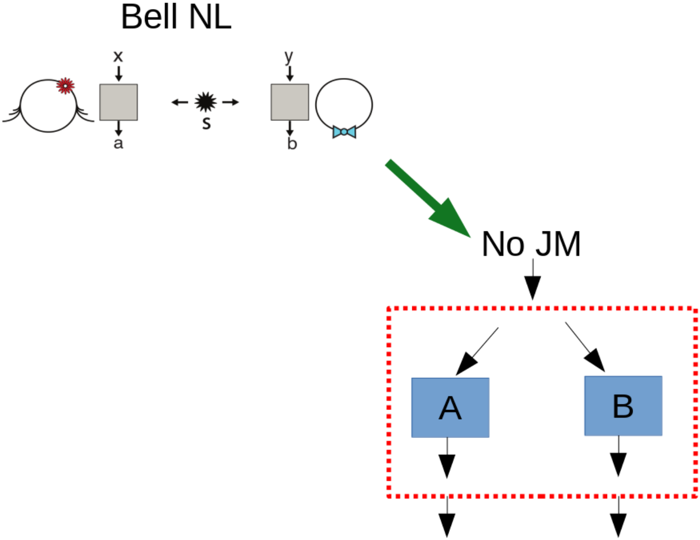
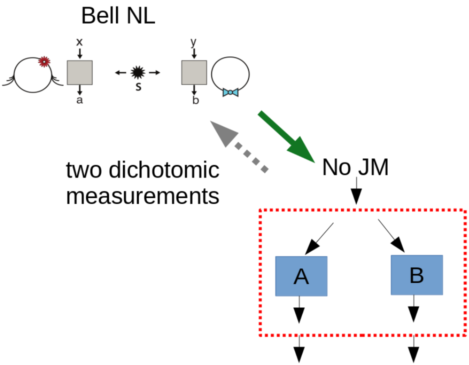


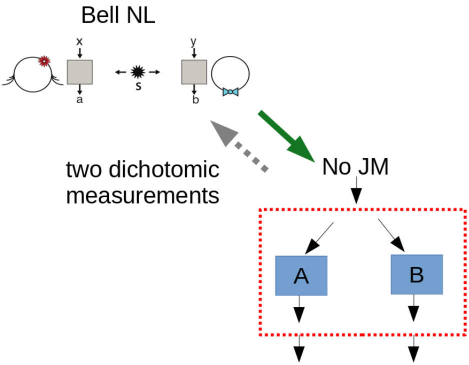
Bell NL and JM



M. Wolf, D. Perez-Garcia, C. Fernandez, PRL (2009)

Bell NL and JM

$\{A_{a|x}\}_{a,x=1}^{2}$ not JM $\implies$ $\exists \rho_{AB}$ and $\{B_{b|y}\}$ such that:
$p(ab|xy) = \mathrm{tr}\big(\rho_{AB}\, A_{a|x} \otimes B_{b|y}\big)$ is Bell NL



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Bell nonlocality

- Are all incompatible measurements useful for Bell NL?

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  Joint measurability, EPR steering, and Bell nonlocality
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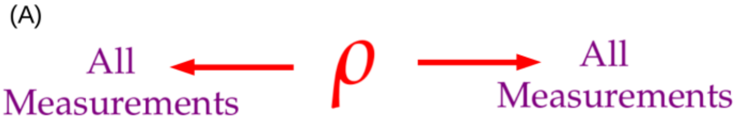
- But, some sets of incompatible measurements are "useless" for Bell NL. . .

Local hidden variable model



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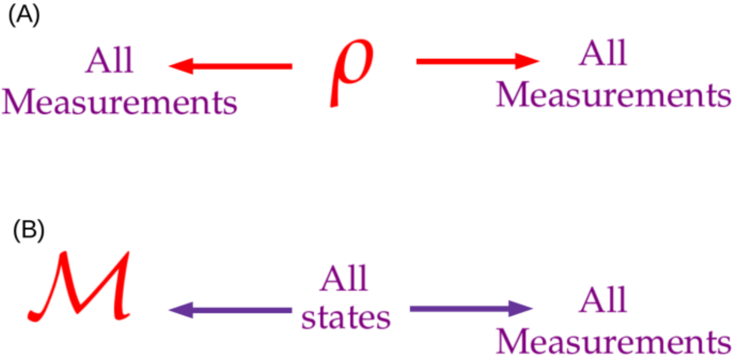

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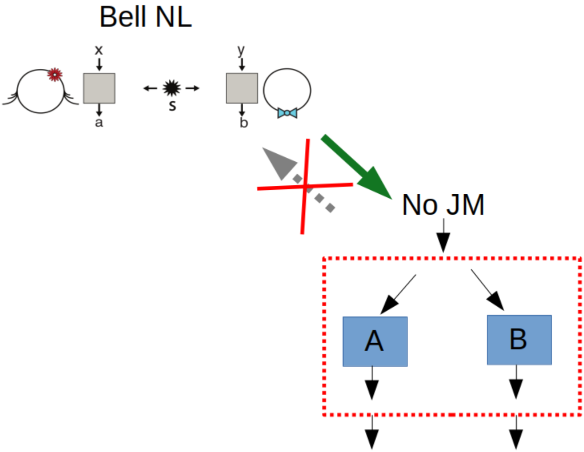
- LHV model for a qubit trine
  Measurement incompatibility does not give rise to Bell violation in general
  E. Bene, T. Vertesi, NJP (2018)

Bell NL and JM



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Bell NL and JM

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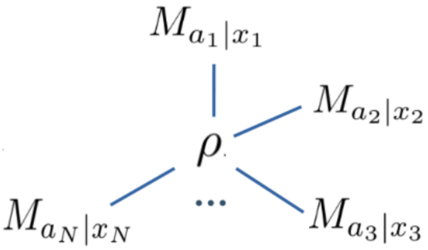
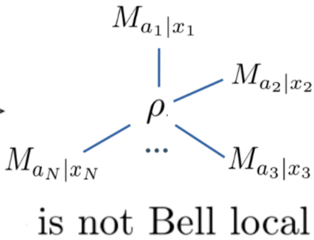
- How about multipartite scenarios?
- How about an N-partite Bell scenario where All parties perform the same measurement



Multipartite Bell NL and JM

$$\{M_{a|x}\} \text{ is not JM} \implies$$



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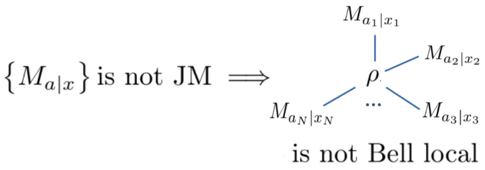
Multipartite Bell NL and JM

If $d = 2$ and $\{M_{a|x}\}$ is not JM, there exists a number of parties N and a state ρ such that

$$p(a_1 \dots a_N | x_1 \dots x_N) = \mathrm{tr}\big(\rho\, M_{a_1|x_1} \otimes \dots \otimes M_{a_N|x_N}\big)$$

is Bell NL



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Applications

- Corollary:
  Let $D_\eta(\rho) := \eta\rho + (1-\eta)\frac{I}{d}$ be the depolarising map. If $\eta > \frac{1}{2}$, there exists a N-partite quantum state ρ_N such that

$$D_\eta^{\otimes N}(\rho_N) = D_\eta \otimes D_\eta \dots D_\eta(\rho_N)$$

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  is Bell NL.
- New JM quantifier for qubits:
  How many parties do you need to display Bell NL ?

Superactivation of Bell JM

- Let $\{A_{a|x}\}$ be a set of incompatible qubit measurements with LHV for bipartite Bell NL

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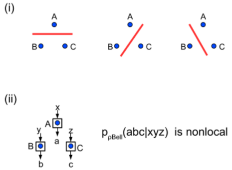
- Anonymous NL:



T.Vertesi, N. Brunner, PRL (2012)
YC Liang, FJ Curchod, J Bowles, N Gisin, PRL (2014)

Proof methods

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- View Bell locality as separability in Generalised Probabilistic Theories (GPT). $p(ab|xy) = \sum_\lambda \pi(\lambda) p_A(a|x\lambda) p_B(b|y\lambda)$
- $\mathcal{M}$ is "entanglement annihilating" iff $\mathcal{M}^{\otimes N}(\rho) = \mathrm{tr}\big(\rho\, M_{a_1|x_1} \otimes \ldots \otimes M_{a_N|x_N}\big)$ is separable (Bell local) $\forall \rho$ and $\forall N \in \mathbb{N}$.

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- $\mathcal{M}$ is "entanglement annihilating" iff $\mathcal{M}^{\otimes N}(\rho) = \mathrm{tr}\big(\rho\, M_{a_1|x_1} \otimes \ldots \otimes M_{a_N|x_N}\big)$ is separable (Bell local) $\forall \rho$ and $\forall N \in \mathbb{N}$.
- Generalise a result on entanglement breaking channels and entanglement annihilating channels.
  When do composed maps become entanglement breaking?
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Proof methods

- View sets of quantum measurements as maps transforming states into probabilities $\mathcal{M}(\rho) = \{\mathrm{tr}\big(\rho M_{a|x}\big)\}_{ax}$
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- Recognise that, measurements lead to Bell local correlations of qubit states iff they're generalising annihilating channels.

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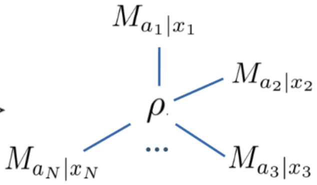
- Simple/useful criteria for measurement Bell NL?

Thank you!

$\{M_{a|x}\}$ is not JM $\implies$



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