Higher-Order Quantum Operations

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CEQIP

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What are higher-order quantum operations?

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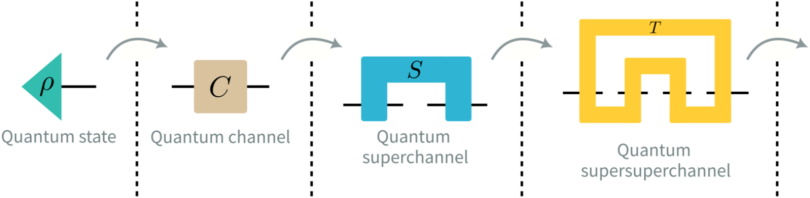
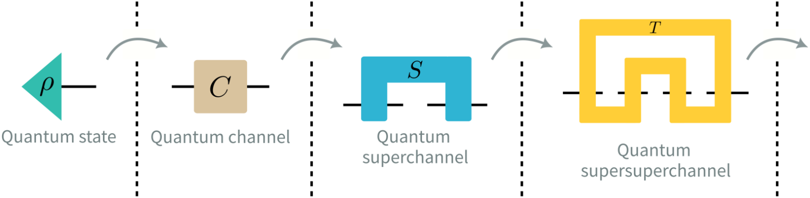
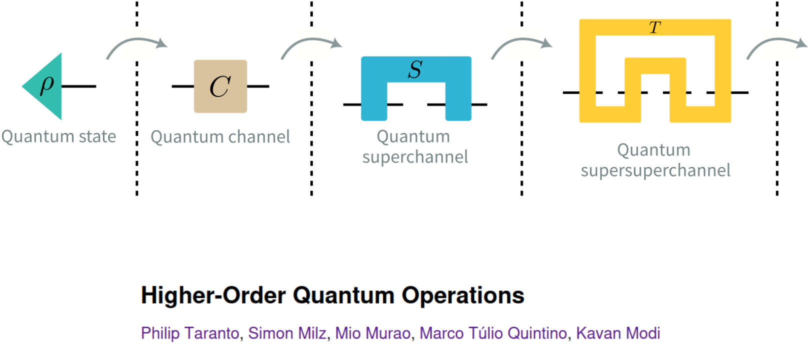


What are higher-order quantum operations?



Why, how, and when to use higher-order operations?

What are higher-order quantum operations?



Why, how, and when to use higher-order operations?

Presentation outline

- Warming up: Transforming quantum states

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- Part 1: Transforming quantum operations

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- Part 2: Measuring quantum operations

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- Part 1: Transforming quantum operations
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- Part 3: Transformations beyond the circuit formalism
- Part 4: The cost of a quantum circuit simulation

Warming up:

How to transform quantum states?

Quantum state

$$\rho \in \mathcal{L}(\mathcal{H}) \cong \mathcal{L}(\mathbb{C}^d)$$

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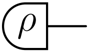

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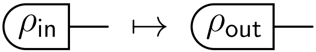

Maths oriented quantum info

- Linear maps:

$$\widetilde{C} : \mathcal{L}(\mathcal{H}_{\text{in}}) \to \mathcal{L}(\mathcal{H}_{\text{out}}),$$

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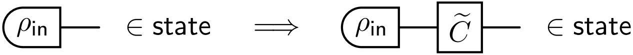
- "Transform quantum states into quantum states"



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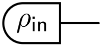
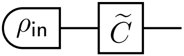


$\in$ state $\quad \Longrightarrow \quad$



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- TP:
$$\text{if } \operatorname{tr}(\rho) = 1, \quad \text{then } \operatorname{tr}\Big(\widetilde{C}(\rho)\Big) = 1$$

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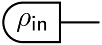

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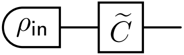

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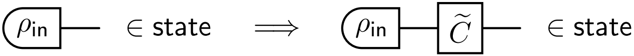

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- OK... but how about state transposition, $\widetilde{T}(\rho) = \rho^T$?

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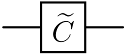
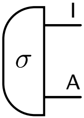


is a quantum channel when



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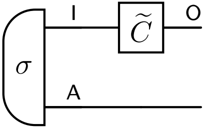

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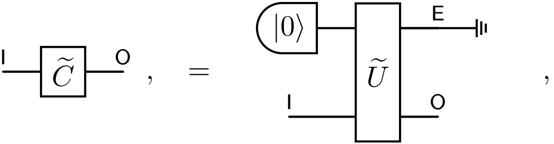

(1)

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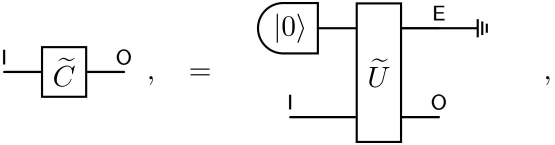

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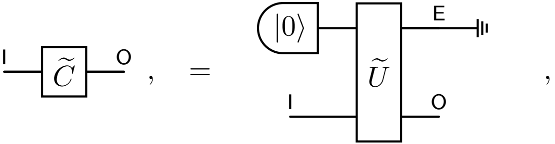

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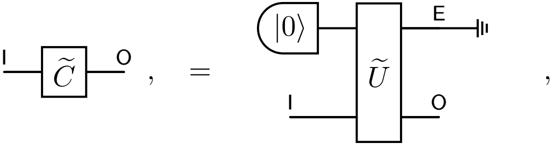

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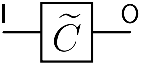
- We're all happy! Pick your favourity approach. :)

Part 1

Trasnforming quantum channels

Quantum channels

$$\widetilde{C} : \mathcal{L}(\mathcal{H}_{\mathsf{I}}) \to \mathcal{L}(\mathcal{H}_O)$$



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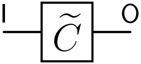
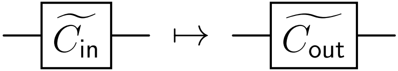


$\widetilde{C}$ is CPTP

How to transform quantum channels?

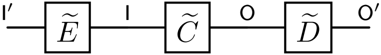
$$\widetilde{C_{\text{in}}} \mapsto \widetilde{C_{\text{out}}}$$



Definitely, this can be done

- Pre-processing and post-processing:

$$\widetilde{\widetilde{S}}(\widetilde{C}) \quad =$$



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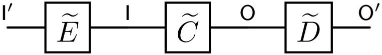
$\tilde{\tilde{S}}(\tilde{C}) =$ 

- As a supermap $\tilde{\tilde{S}} : [\mathcal{L}(\mathcal{H}_{\mathsf{I}}) \to \mathcal{L}(\mathcal{H}_{\mathsf{O}})] \to [\mathcal{L}(\mathcal{H}_{\mathsf{I}'}) \to \mathcal{L}(\mathcal{H}_{\mathsf{O}'})]$

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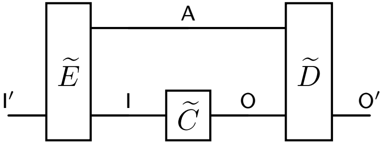

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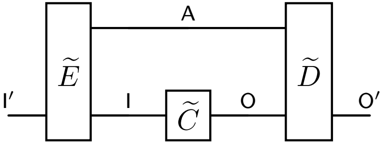

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What can we accept?

- Quantum superchannel:
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$$\text{if } \widetilde{C} \text{ is TP, then } \widetilde{\widetilde{S}}(\widetilde{C}) \text{ is TP}$$

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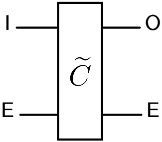
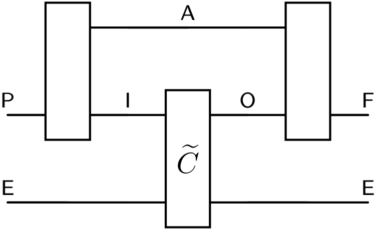
- Well.. Completely CP-preserving



is CP, then



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Quantum superchannels

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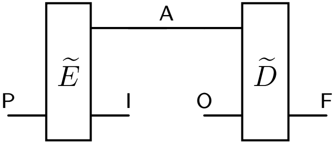
$\widetilde{\widetilde{S}} =$ 

G. Chiribella, G. M. D'Ariano, and P. Perinotti EPL (2008)

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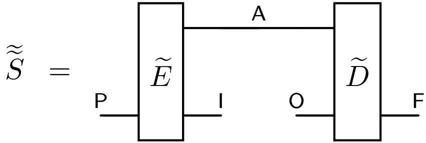

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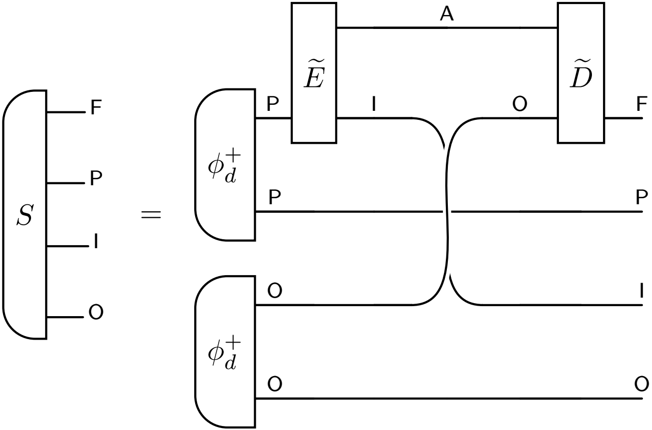
- Causality is proven!

Charactersiting quantum superchannels

Choi isomorphism for supermaps:



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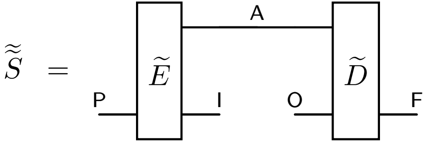

$S \in \mathcal{L}(\mathcal{H}_\mathsf{P} \otimes \mathcal{H}_\mathsf{I} \otimes \mathcal{H}_\mathsf{O} \otimes H_\mathsf{F})$ is a superchannel iff

$$S \geq 0$$

$$\mathrm{tr}_\mathsf{F}(S) = \mathrm{tr}_\mathsf{OF}(S) \otimes \frac{I_\mathsf{F}}{d_\mathsf{F}}$$

$$\mathrm{tr}_\mathsf{IOF}(S) = \mathrm{tr}_\mathsf{PIOF}(S) \otimes \frac{I_\mathsf{I}}{d_\mathsf{I}}$$

$$\mathrm{tr}(S) = d_\mathsf{P} d_\mathsf{O}$$

G. Chiribella, G.M D'Ariano, P. Perinotti, PRL (2008)
G. Gutoski, J. Watrous, STOC (2007)

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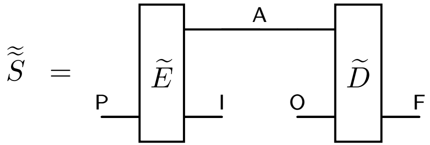

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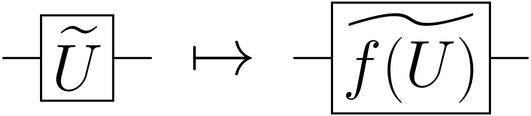
Affine and positive semidefinite constraints $\Longrightarrow$ SDP!!

Quantum unitary transformations



What do we want?

Ideally. . .

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Something like this:

$$-\boxed{\widetilde{Y}}-\boxed{\widetilde{U_2}}-\boxed{\widetilde{Y}}- \;=\; -\boxed{\widetilde{U_2^*}}-$$

Phys. Rev. Research (2019)
J. Miyazaki, A. Soeda, and M. Murao

Applications

- Unitary conjugation, $f(U) = U^*$:
  $F(d) = \frac{2}{d(d-1)}$, or $p(d > 2) = 0$.
  G. Chiribella, D. Ebler, NJP (2016), J. Miyazaki, A. Soeda, M. Murao, PRR (2019)

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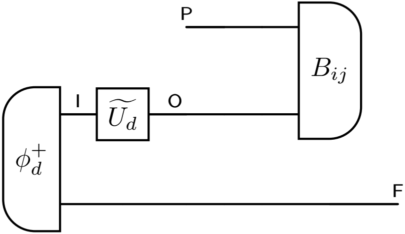
- Unitary Storage-and-Retrieval $f(U) = U$:
  A. Bisio, G. Chiribella, G. M. D'Ariano, S. Facchini, P. Perinotti M. Sedlák, PRA (2010) A. Bisio, M. Ziman, PRL (2019)

Explicit construction for unitary transposition



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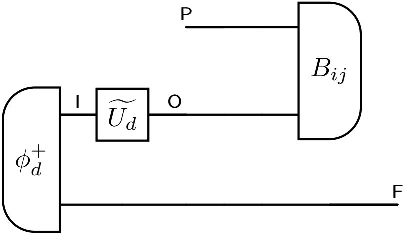

$$(U \otimes \mathbb{1}) \left|\phi_d^+\right\rangle = \left(\mathbb{1} \otimes U^T\right) \left|\phi_d^+\right\rangle$$

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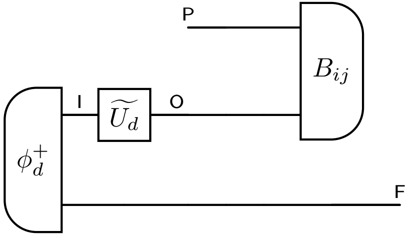
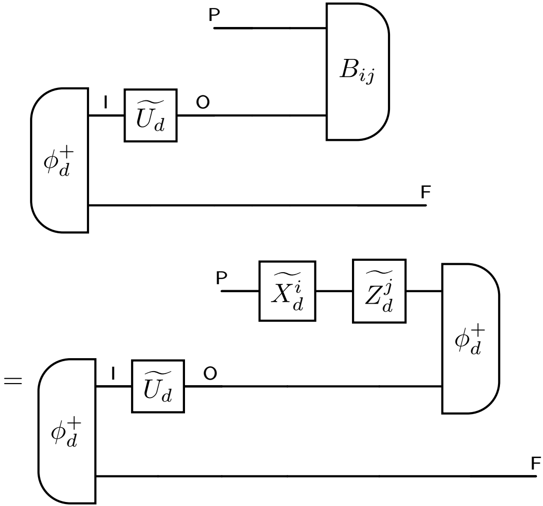


$$(U \otimes \mathbb{1}) \left|\phi_d^+\right\rangle = \left(\mathbb{1} \otimes U^T\right) \left|\phi_d^+\right\rangle$$

$$B_{ij} := \left(Z_d^j X_d^i \otimes \mathbb{1}\right) \left|\phi_d^+\right\rangle$$
$$i, j \in \{0, \ldots, d-1\}$$

Explicit construction for unitary transposition



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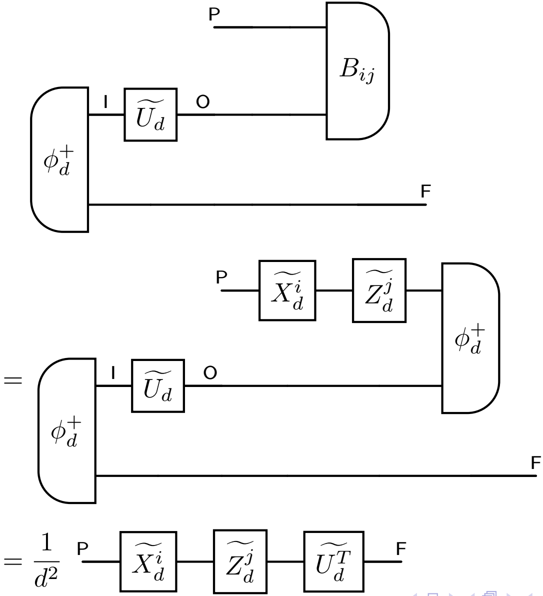
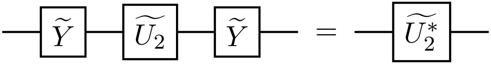


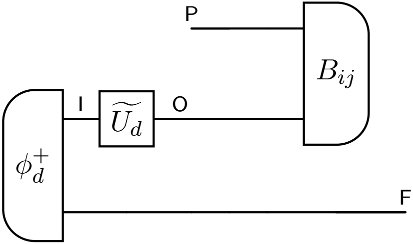
Qubit unitary inversion

For qubits, we have:



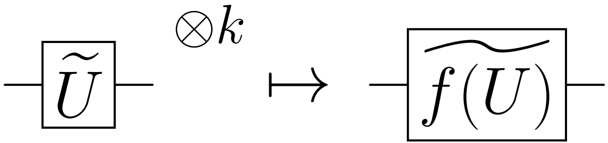
Hence, with $p = 1/4$, we can invert an arbitrary unitary operation!

Delayed input state



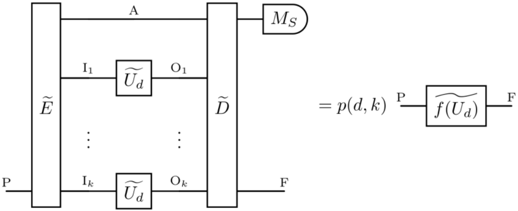
Quantum unitary transformations

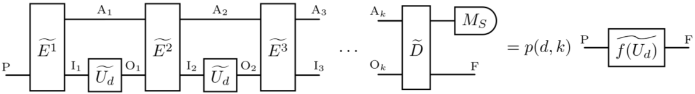
We can call/query operations many times!



Multiple calls

- Scenarios with multiple calls:





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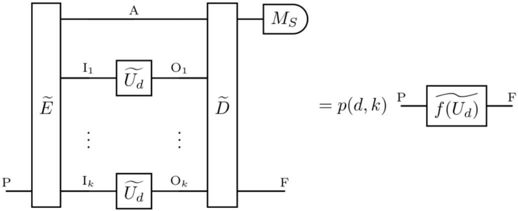

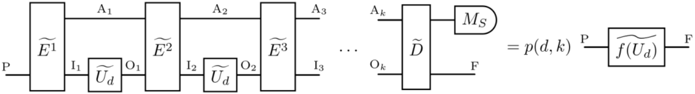

- This also fits the higher-order quantum operations framework

The homomorphic case

- When $f(UV) = f(U)f(V)$, parallel calls are optimal
  A. Bisio, G. M. D'Ariano, P. Perinotti, M. Sedlak, PLA (2014)

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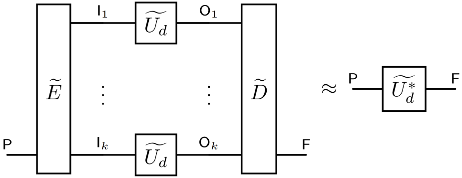

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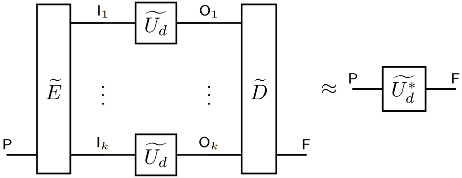

- We can use SDP duality and group theoretic methods to prove that $F(d,k) \leq \frac{k+1}{d(d-k)}$
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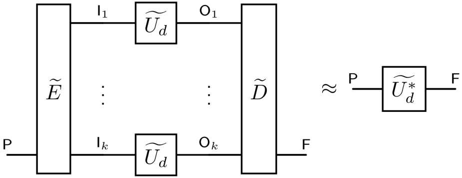

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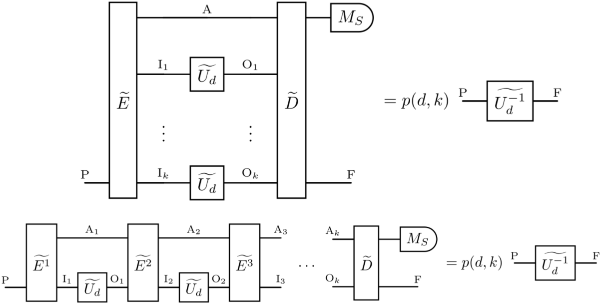
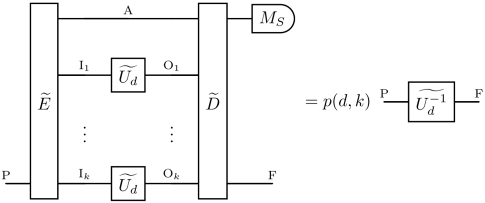
- Also, if $k < d-1$, $p = 0$
  M.T. Quintino, Q. Dong, A. Shimbo, A. Soeda, M. Murao
  PRA (2019)

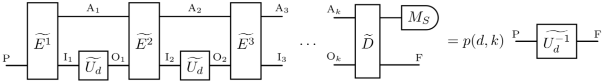
Unitary inversion



- Parallel $(d = 2)$: $p = 1 - \frac{3}{k+3}$

Unitary inversion





- Parallel ($d = 2$): $p = 1 - \frac{3}{k+3}$
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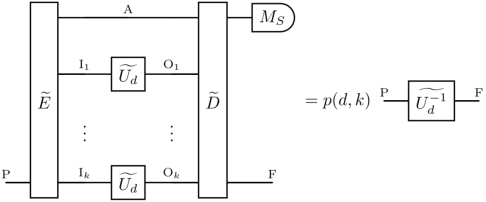
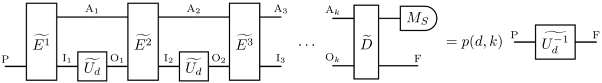
$$1 - \frac{1}{k} \sim 1 - \frac{d^2-1}{\lfloor \frac{k}{d-1} \rfloor + d^2 - 1} \leq p \leq 1 - \frac{d^2-1}{k(d-1)+d^2-1} \sim 1 - \frac{1}{k}$$

Unitary inversion





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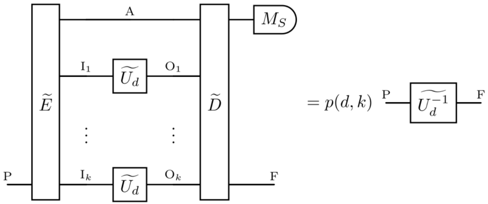

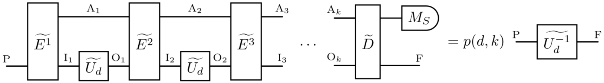

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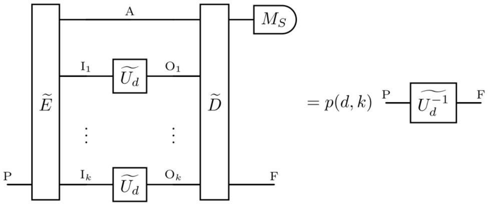
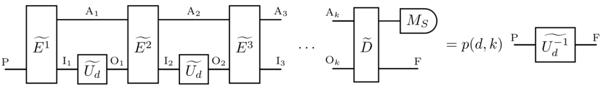
Unitary inversion





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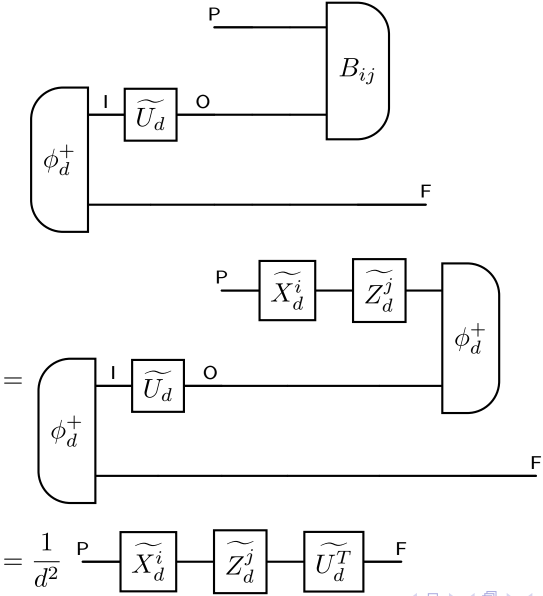
- Optimal parallel $\implies$ delayed input-state
- If $k < d-1$, then $p = 0$
- Sequential ($k \geq d-1$): $p \geq 1 - \left(1 - \frac{1}{d^2}\right)^{\left\lceil \frac{k+2-d}{d} \right\rceil} \sim 1 - \frac{1}{e^k}$

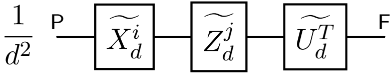
M.T. Quintino, Q. Dong, A. Shimbo, A. Soeda, M. Murao
PRA (2019), PRL (2019)

Unitary transposition



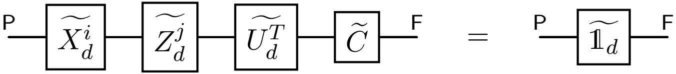
Adaptive circuits

When we fail, we lose track of the unknown input state...
then we cannot re-iterate this protocol...



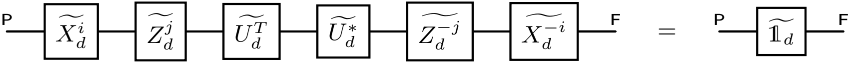
Adaptive circuits

Can we "reset the protocol" when we fail?



Adaptive circuits

Sometimes we can reset it.



Success or draw $\implies$ repeat until success (approaches one exponetially)

Arbitrary functions $f(U_d)$

Nice!
Success or draw strategy exists for inverse and transposition!

Arbitrary functions $f(U_d)$

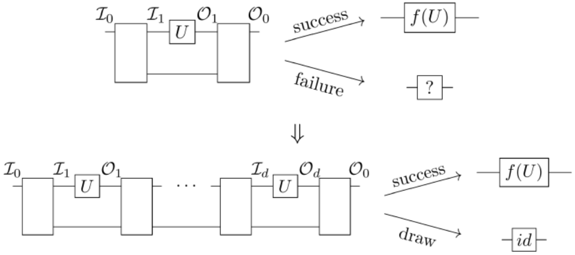
Nice!
Success or draw strategy exists for inverse and transposition!
but how about other functions of unitaries?

$$U_d^{\otimes k} \mapsto f(U_d)$$

Success or draw

Theorem

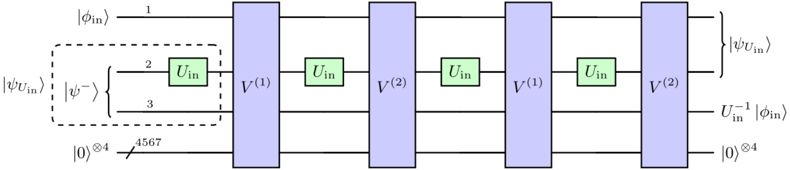
With $k = d$ calls, success or draw is always possible!



Q. Dong, M.T. Quintino, A. Soeda, M. Murao
PRL (2021)

Deterministic and exact is possible

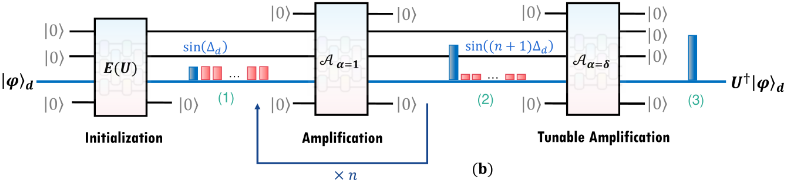
For qubits, $k = 4$ calls are enough:



S. Yoshida, A. Soeda, M. Murao PRL (2023)

Success or draw

And, for qudits, $k \approx d^2$,



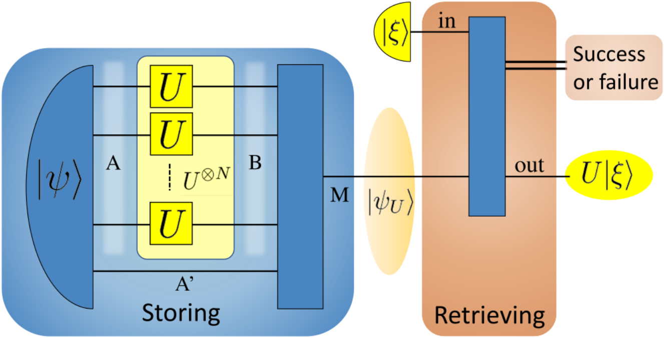
Y. A. Chen, Y. Mo, Y. Liu, L. Zhang, X. Wang (2024)

Probabilistic parallel unitary transposition

Great, and what can we say about parallel probabilistic unitary transposition/inversion?

Probabilistic parallel unitary transposition

Equivalent to unitary SAR[1], can be done with probabilistic PBT

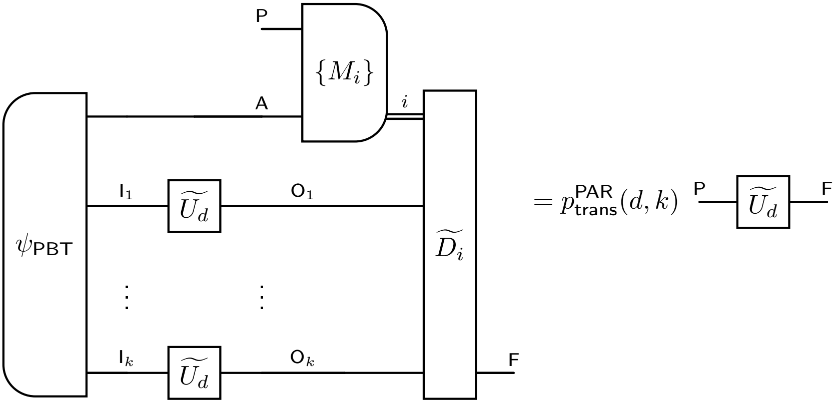


[1] : Optimal Probabilistic Storage and Retrieval of Unitary Channels
M. Sedlák, A. Bisio, and M. Ziman
PRL 2019

Probabilistic parallel unitary transposition

Equivalent to unitary SAR, can be done with probabilistic PBT

$$p^{\mathsf{PAR}}_{\mathsf{trans}}(d,k) = 1 - \frac{d^2-1}{k+d^2-1}, \quad (U_d \otimes \mathbb{1})^{\otimes k} \left|\psi_{\mathsf{PBT}}\right\rangle = \left(\mathbb{1} \otimes U_d^T\right)^{\otimes k} \left|\psi_{\mathsf{PBT}}\right\rangle$$

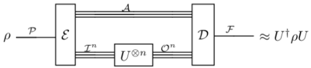


Deterministic parallel unitary inversion and transposition

What does it change in a deterministic non-exact scenario?

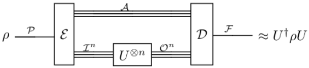
Deterministic parallel unitary inversion and transposition

(d) Parallel unitary inversion



Deterministic parallel unitary inversion and transposition

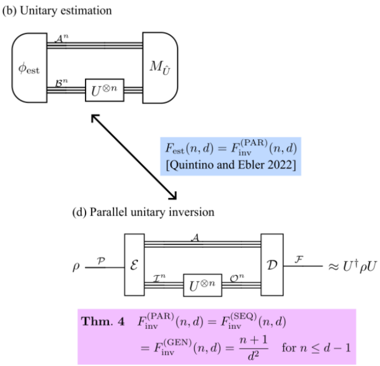
(d) Parallel unitary inversion



Thm. 4 $F_{\text{inv}}^{(\text{PAR})}(n,d) = F_{\text{inv}}^{(\text{SEQ})}(n,d) = F_{\text{inv}}^{(\text{GEN})}(n,d) = \dfrac{n+1}{d^2}$ for $n \leq d-1$

One-to-one Correspondence between Deterministic Port-Based Teleportation and Unitary Estimation
S. Yoshida, Y. Koizumi, M. Studziński, M.T. Quintino, M. Murao
arXiv:2408.11902 (2024)

Deterministic parallel unitary inversion and transposition



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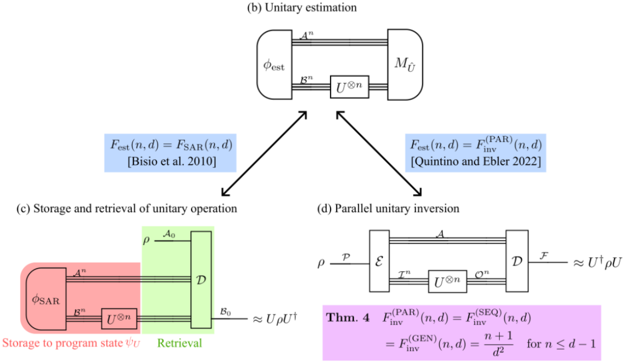

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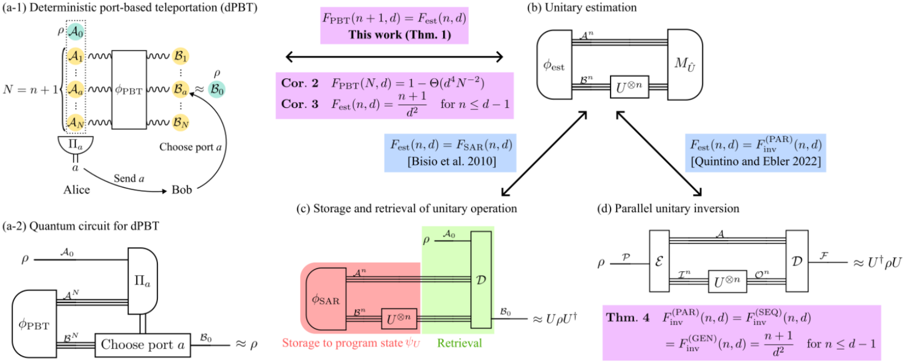

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Part 2

Measuring quantum channels

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- Quantum measurement:

$$\rho \mapsto p_i$$

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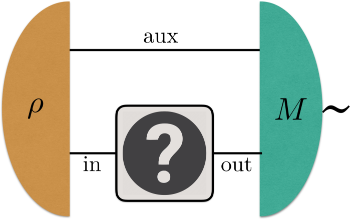
- PPOVM/testers : $T_i \geq 0$, $\sum_i M_i = \sigma \otimes \mathbb{1}$, $\mathrm{tr}(\sigma) = 1$

$$\mathrm{tr}(T_i C) = p_i$$

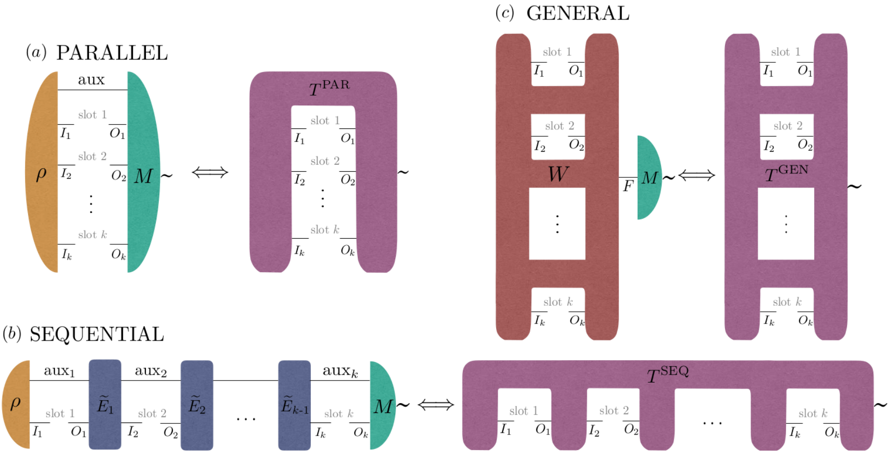
Measuring quantum operations

Super POVMs! (Testers/Process POVMs)



Process POVM: A mathematical framework for the description of process tomography experiments M. Ziman, PRA (2008)
Theoretical framework for quantum networks
G. Chiribella, G.M. D'Ariano, P. Perinotti, PRA (2009)

Measuring quantum operations



G. Chiribella, G.M. D'Ariano, P. Perinotti PRL (2008)
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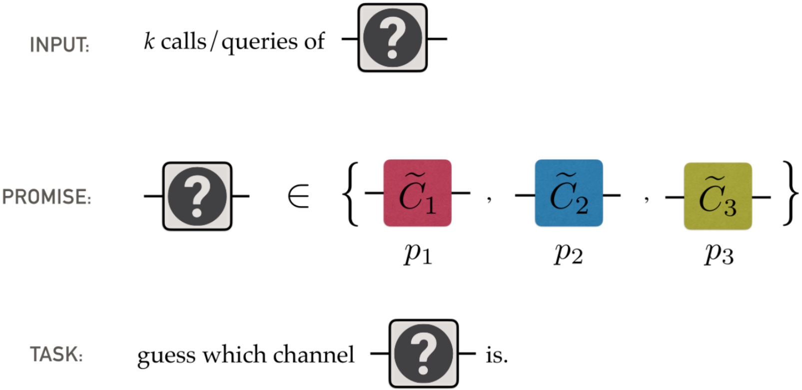
J. Bavaresco, M. Murao, M.T. Quintino PRL (2021)
J. Bavaresco, M. Murao, M.T. Quintino J. Math. Phys. (2022)

Channel discrimination



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  A. Abbott, M. Mhalla, P. Pocreau, PPR (2024)

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- Channel comparison/ identifying malfunctioning gates: A. Soeda, A. Shimbo, M. Murao, PRA (2021), M. Skotiniotis, S. Llorens, R. Hotz, J. Calsamiglia, R. Muñoz-Tapia, PRR (2024)

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- Quantum measurement discrimination: M. Sedlak, M. Ziman, PRA (2014)

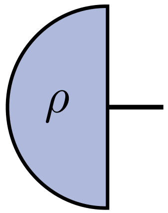
Part 3

Transformations beyond the circuit formalism

Formalism: Higher-order operations

quantum data:

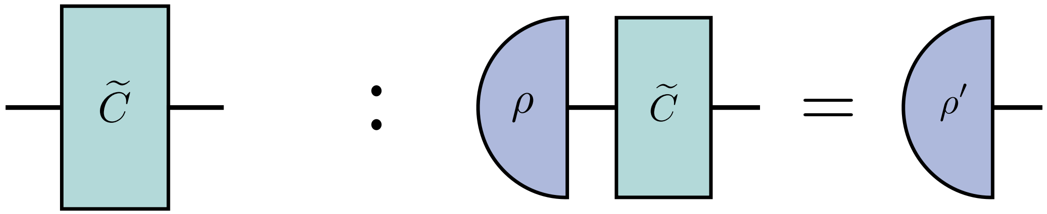
quantum states



$\rho \in \mathcal{L}(\mathcal{H}_{\mathrm{in}})$

quantum functions:

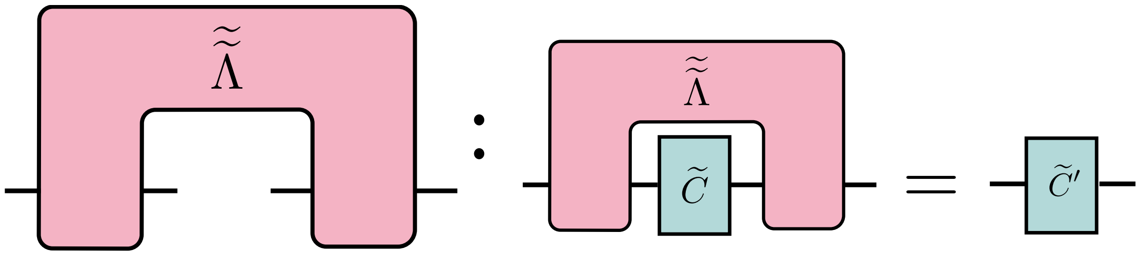
quantum operations
(quantum channels)



$\widetilde{C} : \mathcal{L}(\mathcal{H}_{\mathrm{in}}) \to \mathcal{L}(\mathcal{H}_{\mathrm{out}})$

higher-order quantum operations:

quantum processes



$\widetilde{\widetilde{\Lambda}} : [\mathcal{L}(\mathcal{H}_{\mathrm{in}}) \to \mathcal{L}(\mathcal{H}_{\mathrm{out}})]$
$\to [\mathcal{L}(\mathcal{H}_{\mathrm{in}'}) \to \mathcal{L}(\mathcal{H}_{\mathrm{out}'})]$

"functions of functions"

Key features: Higher-order operations

2) Quantum higher-order operations go beyond the quantum circuit model

Classical	**Quantum**	**Higher-order Quantum**
data (bits)	quantum data (qbits)	quantum data (qbits)
gates	quantum gates	quantum gates
circuits	**circuits**	**quantum circuit structure**

Key features: Higher-order operations

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CLASSICAL	QUANTUM	HIGHER-ORDER QUANTUM
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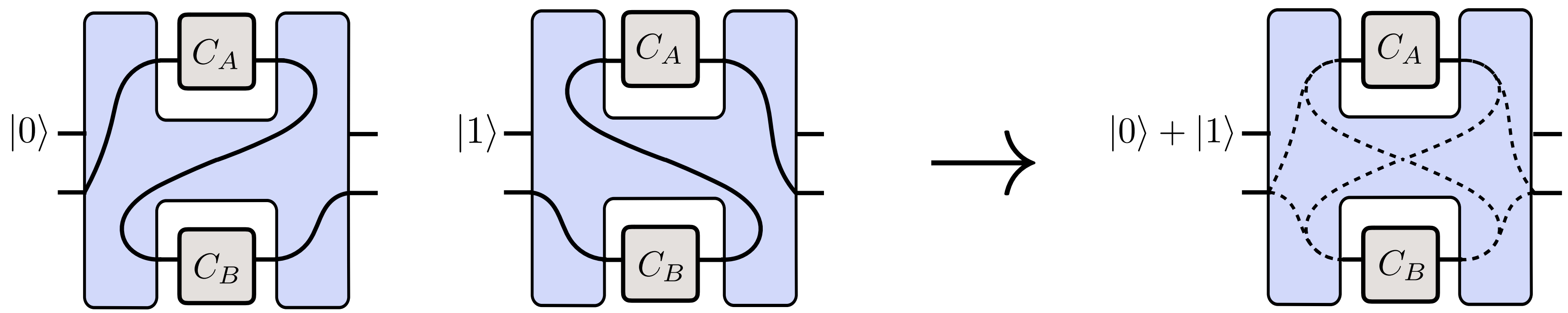
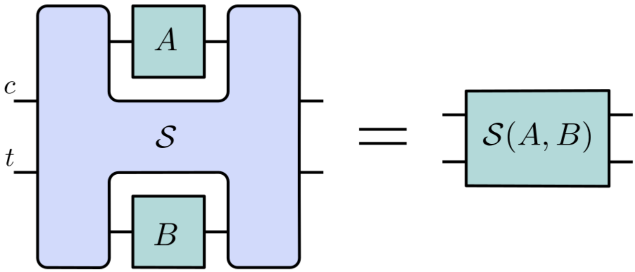
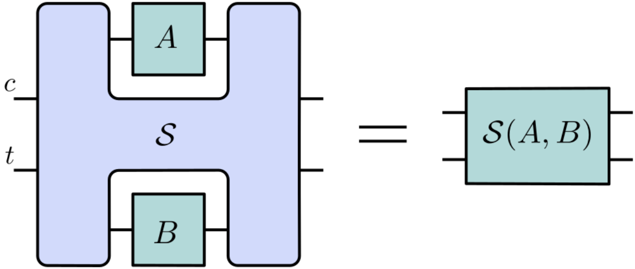


The quantum switch



G. Chiribella, G. M. D'Ariano, P. Perinotti, B. Valiron, PRA (2013)

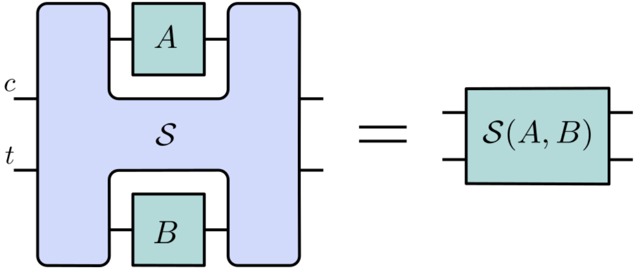
The quantum switch



G. Chiribella, G. M. D'Ariano, P. Perinotti, B. Valiron, PRA (2013)

$$\text{If } A(\rho) = U_A \rho U_A^\dagger \text{ and } B(\rho) = U_B \rho U_B^\dagger$$

The quantum switch



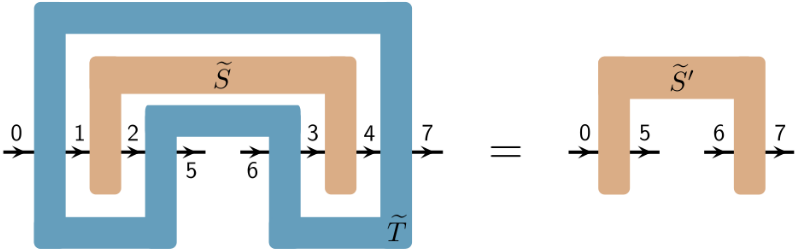
G. Chiribella, G. M. D'Ariano, P. Perinotti, B. Valiron, PRA (2013)

$$\text{If } A(\rho) = U_A \rho U_A^\dagger \text{ and } B(\rho) = U_B \rho U_B^\dagger$$

$$\mathcal{S} : (U_A, U_B) \mapsto |0\rangle\langle 0| \otimes U_B U_A + |1\rangle\langle 1| \otimes U_A U_B$$

Beyond the circuit formalism

The superper channel:

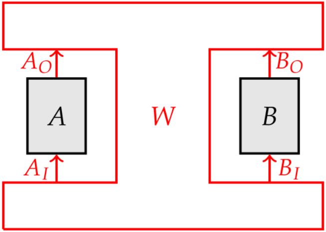


S. Milz, J. Bavaresco, G. Chiribella, Quantum (2022) S. Milz, M.T. Quintino, Quantum (2024)

Beyond the circuit formalism

Process matrices: extracting probabilities from quantum instruments:

$$\mathrm{tr}\big(W A_{a|x} \otimes B_{b|y}\big) = p(ab|xy)$$



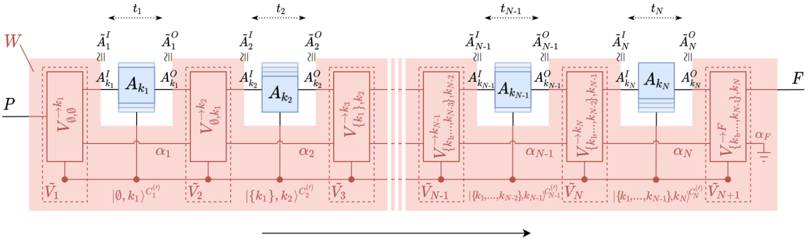
O. Oreshkov, F. Costa, C. Brukner, Nature Communications (2012)
M. Araújo, C. Branciard, F. Costa, A. Feix, C. Giarmatzi, C. Brukner, NJP (2015)

Beyond the circuit formalism

Is indefinite causality OK?

Beyond the circuit formalism

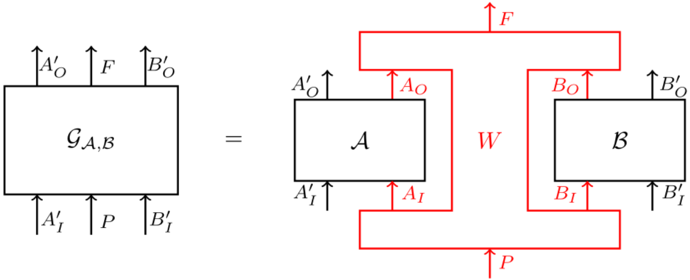
What can be done?
Quantum circuits with quantum control:



J. Wechs, H. Dourdent, A.A. Abbott, C. Branciard, PRX Quantum (2021)

Beyond the circuit formalism

What cannot be done?
Purifiable processs, reversibility preserving:



M. Araújo, A. Feix, M. Navascués, A. Brukner, Quantum (2017)

Part 4

The cost of a quantum circuit simulation

Results based on:

1. arXiv:2409.18420 [pdf, other] quant-ph

Exponential separation in quantum query complexity of the quantum switch with respect to simulations with standard quantum circuits

Authors: Hlér Kristjánsson, Tatsuki Odake, Satoshi Yoshida, Philip Taranto, Jessica Bavaresco, Marco Túlio Quintino, Mio Murao

Abstract: Quantum theory is consistent with a computational model permitting black-box operations to be applied in an indefinite causal order, going beyond the standard circuit model of computation. The quantum switch -- the simplest such example -- has been shown to provide numerous information-processing advantages. Here, we prove that the action of the quantum switch on two n-qubit quantum channels can... ▽ More

Submitted 1 October, 2024; **v1** submitted 26 September, 2024; **originally announced** September 2024.

Comments: 23 pages, 3 figures

2. arXiv:2409.18202 [pdf, other] quant-ph

Can the quantum switch be deterministically simulated?

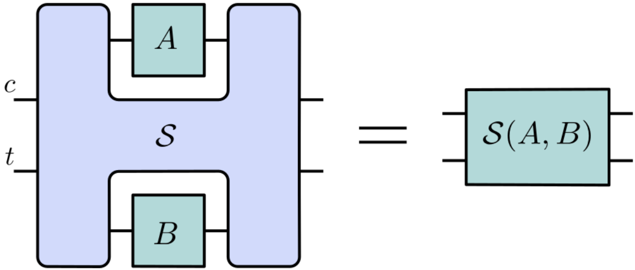
Authors: Jessica Bavaresco, Satoshi Yoshida, Tatsuki Odake, Hlér Kristjánsson, Philip Taranto, Mio Murao, Marco Túlio Quintino

Abstract: Higher-order transformations that act on a certain number of input quantum channels in an indefinite causal order - such as the quantum switch - cannot be described by standard quantum circuits that use the same number of calls of the input quantum channels. However, the question remains whether they can be simulated, i.e., whether their action on their input channels can be deterministically repr... ▽ More

Submitted 26 September, 2024; **originally announced** September 2024.

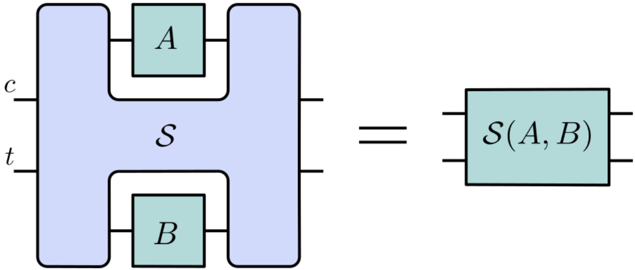
Comments: 16 + 14 pages, 4 + 5 figures

The quantum switch



What can we do with that?

The quantum switch



What can we do with that?

The commuting, anti-commuting game:

Perfect discrimination of no-signalling channels via quantum superposition of causal structures
G. Chiribella, PRA 2012
Witnessing causal nonseparability
M. Araújo, C. Branciard, F. Costa, A. Feix, C. Giarmatzi, Č. Brukner, NJP 2015

Commutation/Anti-Commutation game

(U_A, U_B) is a pair of unitary that

$$U_A U_B = U_B U_A \quad \text{or} \quad U_A U_B = -U_B U_A$$

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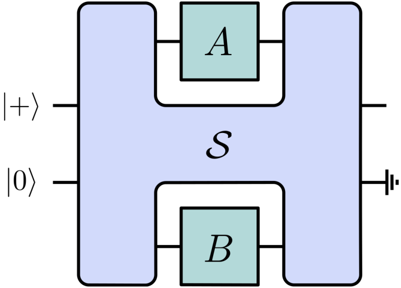
The quantum switch is useful:



Commutation/Anti-Commutation game

- Let $\{(U_A^i, U_B^i)\}_{i=1}^N$ be a set of unitaries that commutes or anticommutes.

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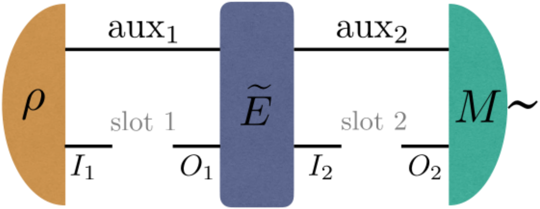
- Standard ordered strategy:



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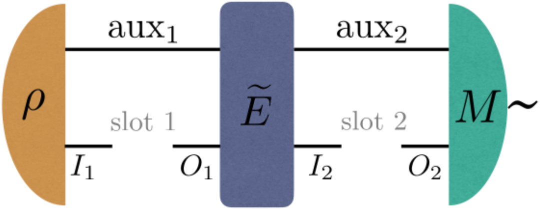

- We can find finite sets of unitaries such that $p_{\text{ordered}} \leq 0.87$.

Commutation/Anti-Commutation game

- Great, the quantum switch is useful!

Commutation/Anti-Commutation game

- Great, the quantum switch is useful!
- But . . .

Commutation/Anti-Commutation game

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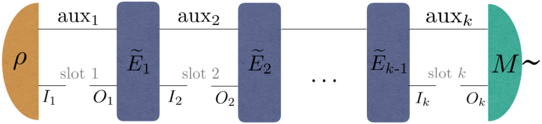
- But ...
- How big is this advantage? What if we do not have the quantum switch, but we have access to more queries?

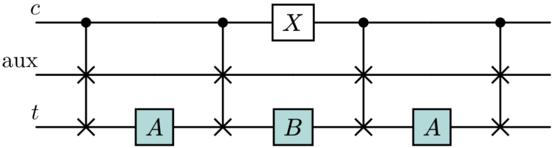
Commutation/Anti-Commutation game

- Great, the quantum switch is useful!
- But ...
- How big is this advantage? What if we do not have the quantum switch, but we have access to more queries?
- With a single extra query, sequential strategies can decide if (U_A, U_B) commutes or anti-commutes



Quantum switch circuit simulation

- Quantum computations without definite causal structure
  G. Chiribella, G. M. D'Ariano, P. Perinotti, B. Valiron, PRA (2013)



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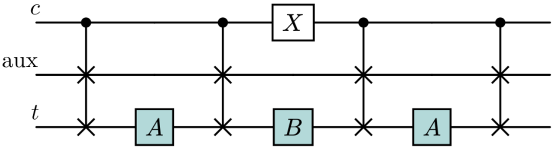
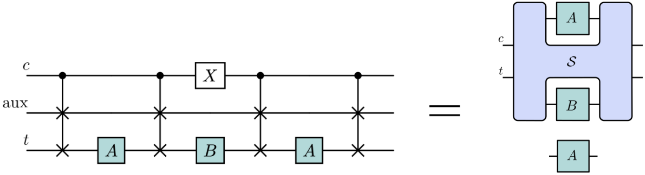


- If A and B are unitary:



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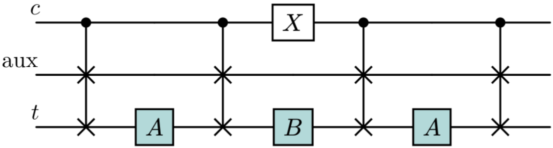

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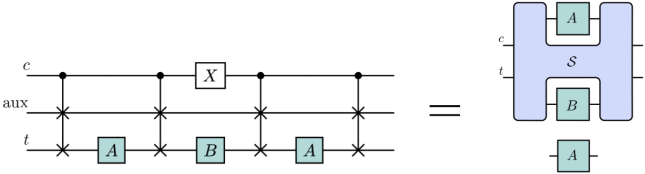


- The switch is essentially useless for query complexity tasks. . .

Quantum switch circuit simulation

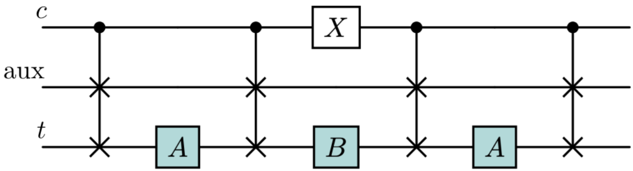
- Wait. . .

Quantum switch circuit simulation

- Wait. . .
- What if the operations are not unitary?

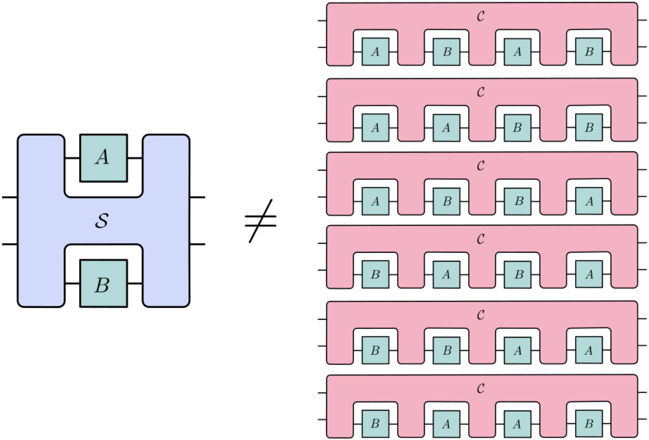
Quantum switch circuit simulation

- Wait. . .
- What if the operations are not unitary?
- E.g., $A(\rho) = B(\rho) = \mathrm{tr}(\rho)\,\frac{\mathbb{I}}{d}$



Quantum switch circuit simulation

Thm1: There is no quantum circuit that simulates the quantum switch when one extra query of each channel is available.



Quantum switch circuit simulation

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Quantum switch circuit simulation

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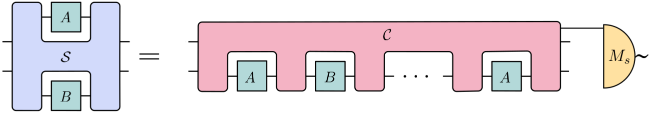
- Probabilistic simulation:



Quantum switch circuit simulation

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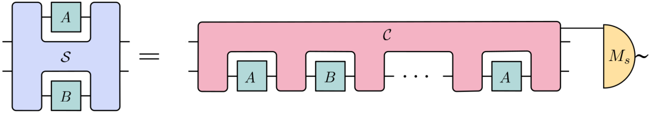
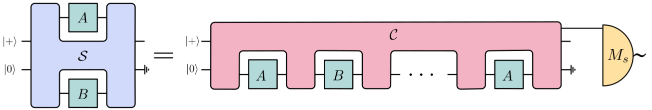
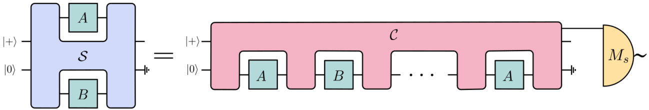


- Restricted probabilistic simulation:



Quantum switch circuit simulation

- How about the probabilities?



Quantum switch circuit simulation

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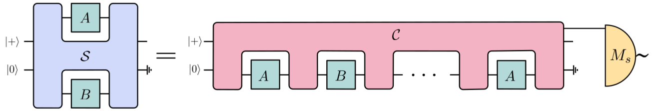
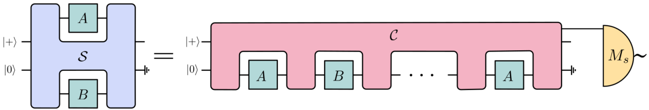
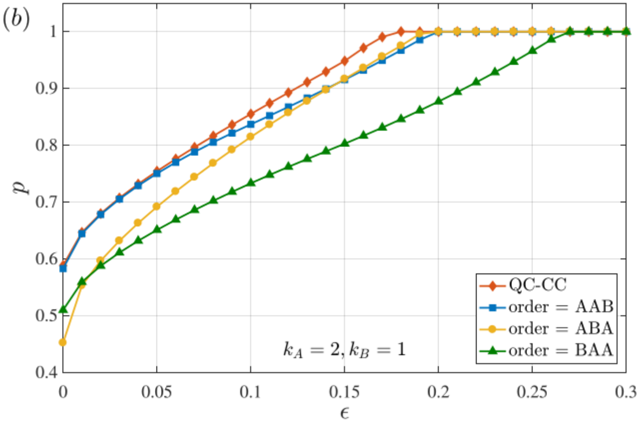


(k_A, k_B)	order	probability
$(1, 1)$	AB	$p < \frac{4001}{10000}$
$(2, 1)$	AAB	$p < \frac{5715}{10000}$
	ABA	$p < \frac{4919}{10000}$
	BAA	$p < \frac{5001}{10000}$

Quantum switch circuit simulation

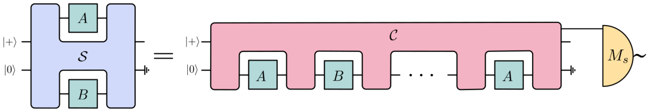
- The result is also robust, $F(\mathcal{S}, \mathcal{S}_{\text{sim}}) = 1 - \epsilon$





Quantum switch circuit simulation

- How about the probabilities when $k = 4$?



Quantum switch circuit simulation

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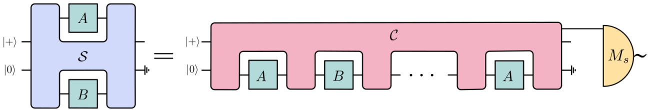
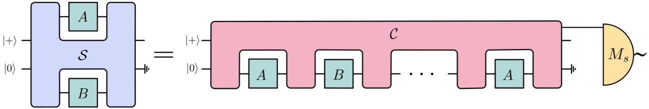


(2, 2)	AABB	$p < \frac{8307}{10000}$
	ABAB	$p < \frac{8484}{10000}$
	ABBA	$p < \frac{8695}{10000}$
(3, 1)	AAAB	$p < \frac{8373}{10000}$
	AABA	$p < \frac{6909}{10000}$
	ABAA	$p < \frac{7597}{10000}$
	BAAA	$p < \frac{6845}{10000}$

Quantum switch circuit simulation

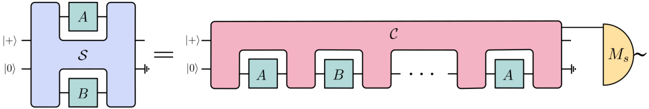
- How about identical channels?



k	order	probability
2	AA	$p < \frac{4001}{10000}$
3	AAA	$p < \frac{6534}{10000}$
4	AAAA	$p = 1$ (*)

Quantum switch circuit simulation

- How about identical channels?



k	order	probability
2	AA	$p < \frac{4001}{10000}$
3	AAA	$p < \frac{6534}{10000}$
4	AAAA	$p = 1$ (*)

If the target is not discarded, $p < 0.942$ for the AAAA order

Quantum switch circuit simulation

- How about unitary channels?

(k_A, k_B)	order (unitary only)	probability
$(1,1)$	AB	$p \approx 0.400$
$(2,1)$	AAB	$p \approx 0.596$
	ABA	$\boldsymbol{p=1}$
	BAA	$p \approx 0.607$
$(2,2)$	AABB	$p = 1$ (*)
	ABAB	$\boldsymbol{p=1}$
	ABBA	$\boldsymbol{p=1}$
$(3,1)$	AAAB	$p \approx 0.708$
	AABA	$\boldsymbol{p=1}$
	ABAA	$\boldsymbol{p=1}$
	BAAA	$p = 1$ (*)

Quantum switch circuit simulation

- How about unitary channels?

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	AABA	$\boldsymbol{p = 1}$
	ABAA	$\boldsymbol{p = 1}$
	BAAA	$p = 1$ (*)

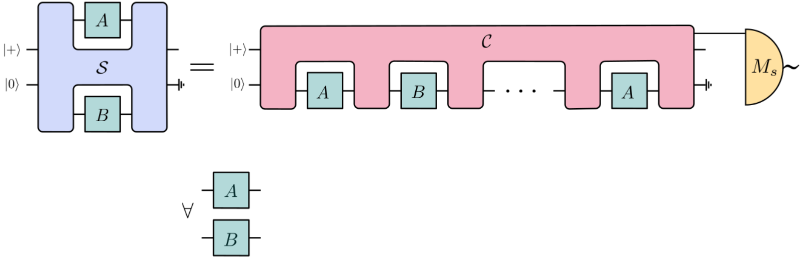
If the target is not discarded, $p < 0.822$ for the AABB order and $p < 0.667$ for the BAAA order

Quantum switch circuit simulation

- How these results were obtained?

Quantum switch circuit simulation

- How these results were obtained?
- Optimise over all inputs:



Quantum switch circuit simulation

- How these results were obtained?
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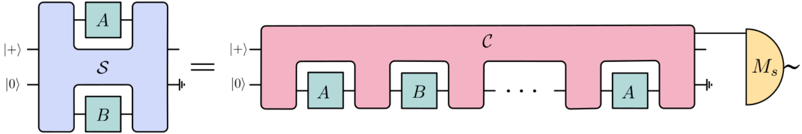
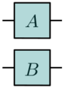


$\forall$ 

$\iff$ $\forall$

$\{A_i\}_i$ that forms a basis for the space spanned by k_A copies of a channel

$\{B_j\}_j$ that forms a basis for the space spanned by k_B copies of a channel

Quantum switch circuit simulation

- SDP (using splitting conic solver)

$$
\begin{aligned}
\textbf{given} \quad & \{J_i^A\}_i, \{J_j^B\}_j, k_A, k_B \\
\textbf{max} \quad & p \\
\textbf{s.t.} \quad & C_s * [(J_i^A)^{\otimes k_A} \otimes (J_j^B)^{\otimes k_B}] = p\, S * (J_i^A \otimes J_j^B) \;\; \forall i,j \\
& C_s \geq 0, \quad C - C_s \geq 0, \\
& \mathbb{P}(C) = C, \quad \mathrm{tr}(C) = d_{c_I} d_{t_I} d_{A_O}^{k_A} d_{B_O}^{k_B},
\end{aligned}
$$

$$
\begin{aligned}
\textbf{given} \quad & \{J_i^A\}_i, \{J_j^B\}_j, k_A, k_B \\
\textbf{min} \quad & \frac{1}{d_{A_I}^{k_A} d_{B_I}^{k_B} d_{c_O} d_{t_O}} \mathrm{tr}(\Gamma) \\
\textbf{s.t.} \quad & \sum_{i,j} \mathrm{tr}\big[R_{ij}\,(S * (J_i^A \otimes J_j^B))\big] = 1 \\
& \Gamma - \sum_{i,j} R_{ij} \otimes [(J_i^A)^{\otimes k_A} \otimes (J_j^B)^{\otimes k_B}]^T \geq 0 \\
& \Gamma \geq 0, \quad \overline{\mathbb{P}}(\Gamma) = \Gamma,
\end{aligned}
$$

any feasible point that yields some
$p < 1$
constitutes a valid upper bound

Quantum switch circuit simulation

- But... is that a mathematical proof?

Quantum switch circuit simulation

- But. . . is that a mathematical proof?
- No! But, we can extract a proof out of it!

Quantum switch circuit simulation

- But. . . is that a mathematical proof?
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Algorithm:

1. Construct symbolic non-floating point operators Γ^{sym} and R^{sym}_{ij} by truncating them and obtaining a symbolic operator with only rational numbers.

2. Force the operators Γ^{sym} and R^{sym}_{ij} to be self-adjoint by making use of the expression $(M+M^\dagger)/2$, which is self-adjoint for any M.

3. Evaluate $t^{\mathrm{sym}} := \sum_{i,j} \mathrm{tr}\left[R^{\mathrm{sym}}_{ij}\,(S * (J^A_i \otimes J^B_j))\right]$, where S, J^A_i, and J^B_j are also symbolic operators. Define $R^{\mathrm{ok}}_{ij} := R^{\mathrm{sym}}_{ij}/t^{\mathrm{sym}}$ for all i,j.

4. Project Γ^{sym} onto the appropriate subspace to obtain $\overline{\mathbb{P}}(\Gamma^{\mathrm{sym}})$.

5. Find $\eta \in \mathbb{R}$ such that $\Gamma^{\mathrm{ok}} := \overline{\mathbb{P}}(\Gamma^{\mathrm{sym}}) + \eta\mathbb{1} \geq 0$ and $\Gamma^{\mathrm{ok}} - \sum_{i,j} R^{\mathrm{ok}}_{ij} \otimes (J_i^{A^{\otimes k_A}} \otimes J_j^{B^{\otimes k_B}})^T \geq 0$

6. Output the quantity $\mathrm{tr}(\Gamma^{\mathrm{ok}})/d_{c_I} d_{t_I} d_{A_O}^{k_A} d_{B_O}^{k_B}$, which is a rigorous upper bound of the primal problem.

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  In particular, its eigendecomposition cannot be compatible with QCQC processes.

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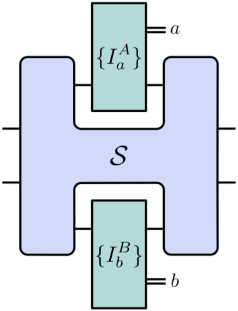
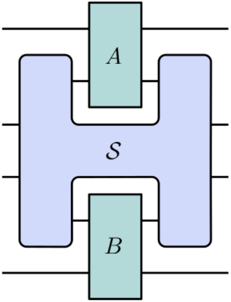
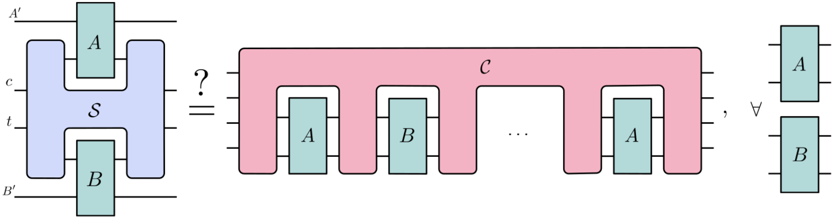
- How about instruments, bipartite channels?





Quantum switch circuit simulation

General simulation:



Quantum switch circuit simulation

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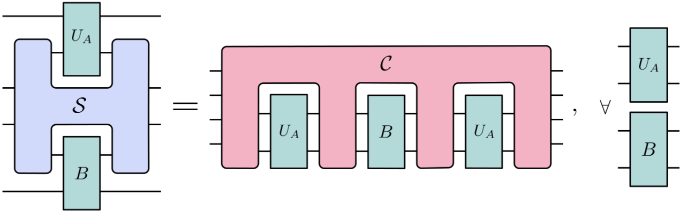

Quantum switch circuit simulation

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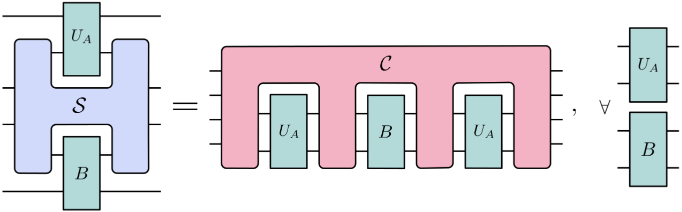
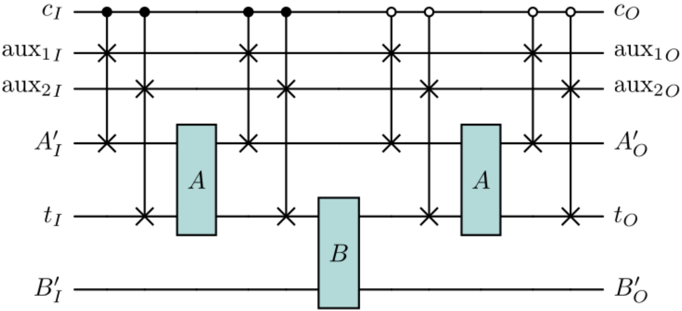


- The circuit:



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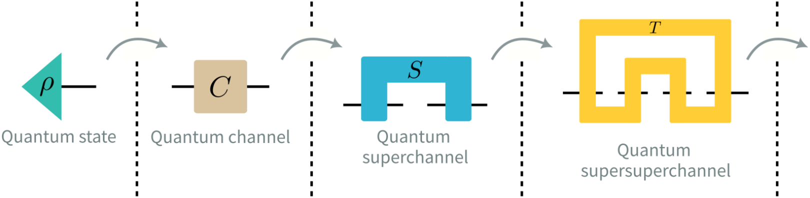
- Channel transformations, channel measurement, channel comparison, oracle based tasks
- What can we say about "gray box" scenarios?
- HOQO methods and approach to adjacent fields?

Thank you


ρ
Quantum state
C
Quantum channel
S
Quantum superchannel
T
Quantum supersuperchannel