Can the quantum switch be deterministically simulated

Jessica Bavaresco, Satoshi Yoshida, Tatsuki Odake, Hlér Kristjánsson, Philip Taranto, Mio Murao, Marco Túlio Quintino

Sorbonne Université, CNRS, LIP6

December 12, 2024

Talk based on:

1. arXiv:2409.18420 [pdf, other] quant-ph

Exponential separation in quantum query complexity of the quantum switch with respect to simulations with standard quantum circuits

Authors: Hlér Kristjánsson, Tatsuki Odake, Satoshi Yoshida, Philip Taranto, Jessica Bavaresco, Marco Túlio Quintino, Mio Murao

Abstract: Quantum theory is consistent with a computational model permitting black-box operations to be applied in an indefinite causal order, going beyond the standard circuit model of computation. The quantum switch -- the simplest such example -- has been shown to provide numerous information-processing advantages. Here, we prove that the action of the quantum switch on two n-qubit quantum channels can... ▽ More

Submitted 1 October, 2024; **v1** submitted 26 September, 2024; **originally announced** September 2024.

Comments: 23 pages, 3 figures

2. arXiv:2409.18202 [pdf, other] quant-ph

Can the quantum switch be deterministically simulated?

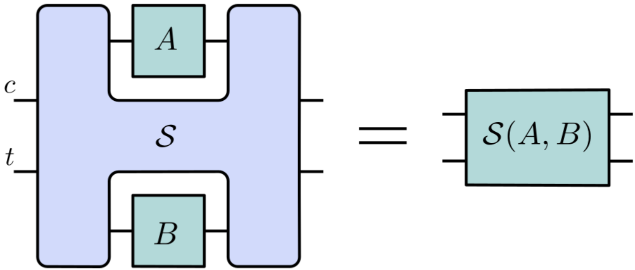
Authors: Jessica Bavaresco, Satoshi Yoshida, Tatsuki Odake, Hlér Kristjánsson, Philip Taranto, Mio Murao, Marco Túlio Quintino

Abstract: Higher-order transformations that act on a certain number of input quantum channels in an indefinite causal order - such as the quantum switch - cannot be described by standard quantum circuits that use the same number of calls of the input quantum channels. However, the question remains whether they can be simulated, i.e., whether their action on their input channels can be deterministically repr... ▽ More

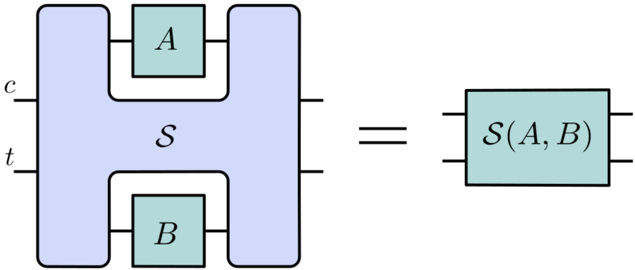
Submitted 26 September, 2024; **originally announced** September 2024.

Comments: 16 + 14 pages, 4 + 5 figures

The quantum switch

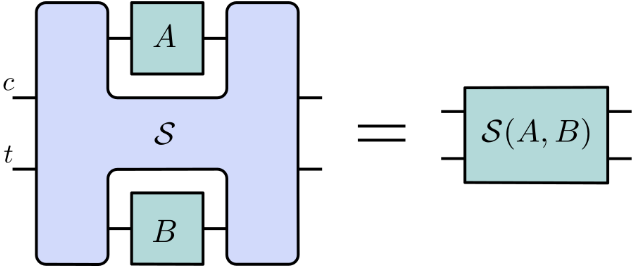


The quantum switch



If $A(\rho) = U_A \rho U_A^\dagger$ and $B(\rho) = U_B \rho U_B^\dagger$

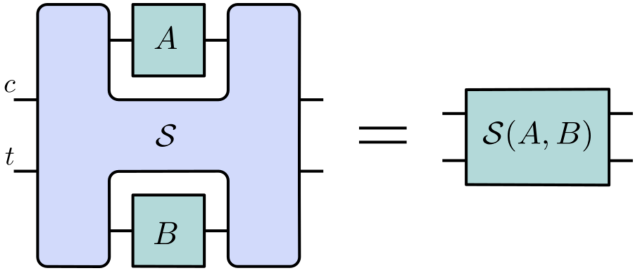
The quantum switch



If $A(\rho) = U_A \rho U_A^\dagger$ and $B(\rho) = U_B \rho U_B^\dagger$

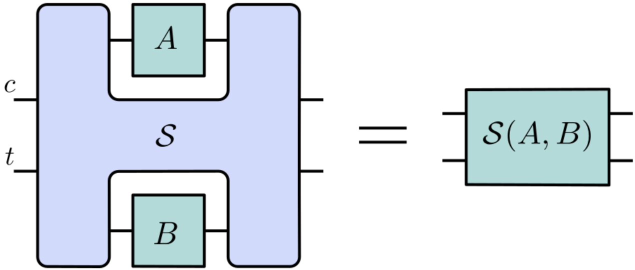
$$\mathcal{S} : (U_A, U_B) \mapsto |0\rangle\langle 0| \otimes U_B U_A + |1\rangle\langle 1| \otimes U_A U_B$$

The quantum switch



What can we do with that?

The quantum switch



What can we do with that?

The commuting, anti-commuting game:

Perfect discrimination of no-signalling channels via quantum superposition of causal structures
G. Chiribella, PRA 2012
Witnessing causal nonseparability
M. Araújo, C. Branciard, F. Costa, A. Feix, C. Giarmatzi, Č. Brukner, NJP 2015

Commutation/Anti-Commutation game

(U_A, U_B) is a pair of unitary that

$$U_A U_B = U_B U_A \quad \text{or} \quad U_A U_B = -U_B U_A$$

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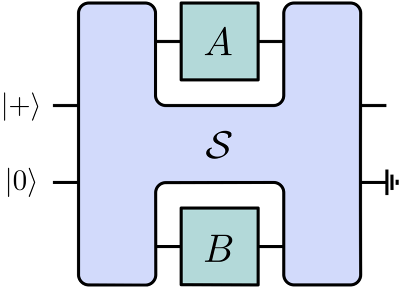
The quantum switch is useful:



Commutation/Anti-Commutation game

- Let $\{(U_A^i, U_B^i)\}_{i=1}^N$ be a set of unitaries that commutes or anticommutes.

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- Given a pair of unitaries at random, can you decide if they commute or anticommute?

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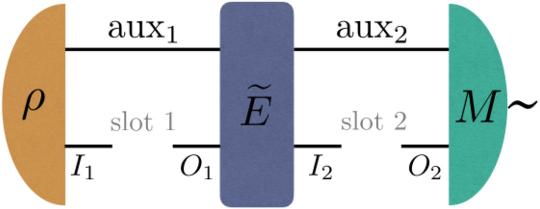
- Standard ordered strategy:



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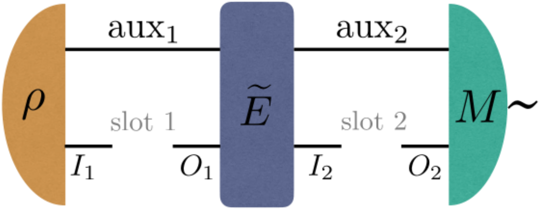


- We can find finite sets of unitaries such that $p_{\text{ordered}} \leq 0.87$.

Commutation/Anti-Commutation game

- Great, the quantum switch is useful!

Commutation/Anti-Commutation game

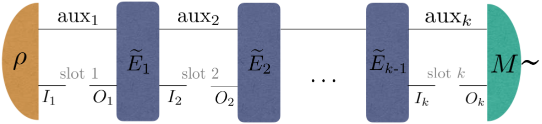
- Great, the quantum switch is useful!
- But . . .

Commutation/Anti-Commutation game

- Great, the quantum switch is useful!
- But . . .
- How big is this advantage? What if we do not have the quantum switch, but we have access to more queries?

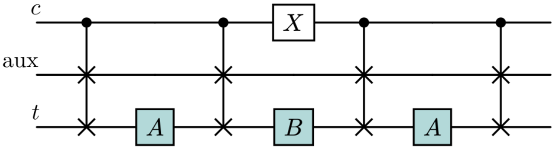
Commutation/Anti-Commutation game

- Great, the quantum switch is useful!
- But ...
- How big is this advantage? What if we do not have the quantum switch, but we have access to more queries?
- With a single extra query, sequential strategies can decide if (U_A, U_B) commutes or anti-commutes



Quantum switch circuit simulation

- Quantum computations without definite causal structure
  G. Chiribella, G. M. D'Ariano, P. Perinotti, B. Valiron, PRA (2013)



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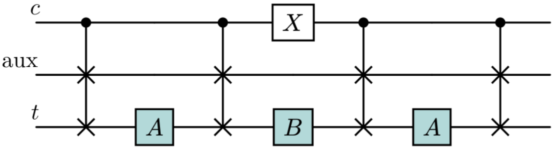
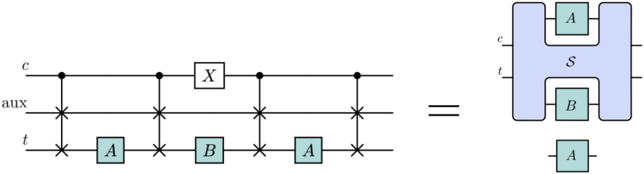


- If A and B are unitary:



Quantum switch circuit simulation

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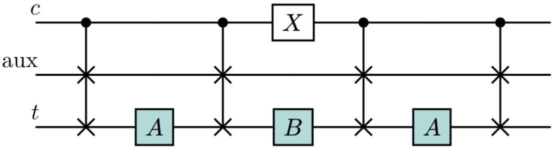

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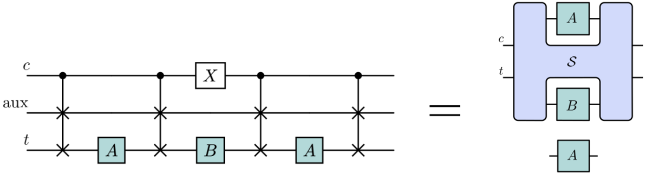


- The switch is essentially useless for query complexity tasks. . .

Quantum switch circuit simulation

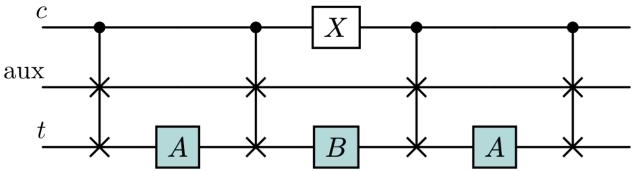
- Wait. . .

Quantum switch circuit simulation

- Wait. . .
- What if the operations are not unitary?

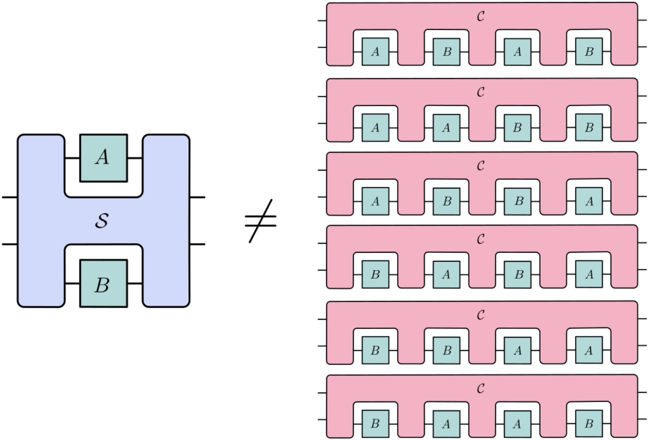
Quantum switch circuit simulation

- Wait. . .
- What if the operations are not unitary?
- E.g., $A(\rho) = B(\rho) = \mathrm{tr}(\rho)\,\frac{\mathbb{I}}{d}$



Quantum switch circuit simulation

Thm1: There is no quantum circuit that simulates the quantum switch when one extra query of each channel is available.



Quantum switch circuit simulation

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Quantum switch circuit simulation

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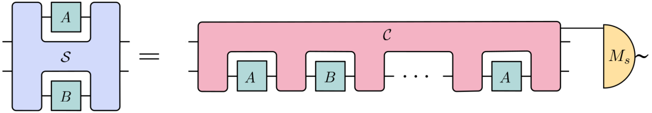
- Probabilistic simulation:



Quantum switch circuit simulation

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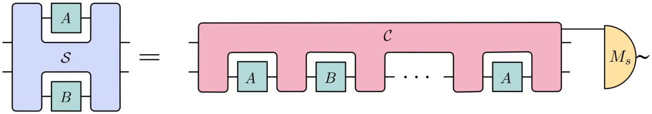
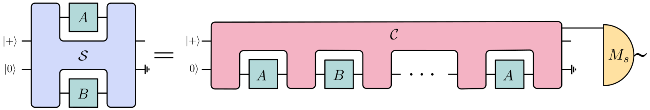
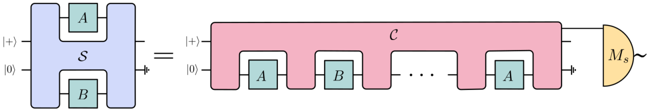


- Restricted probabilistic simulation:



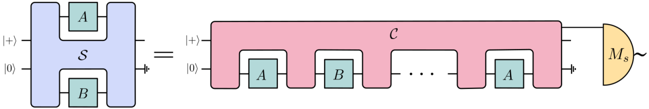
Quantum switch circuit simulation

- How about the probabilities?



Quantum switch circuit simulation

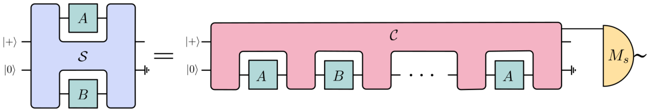
- How about the probabilities?

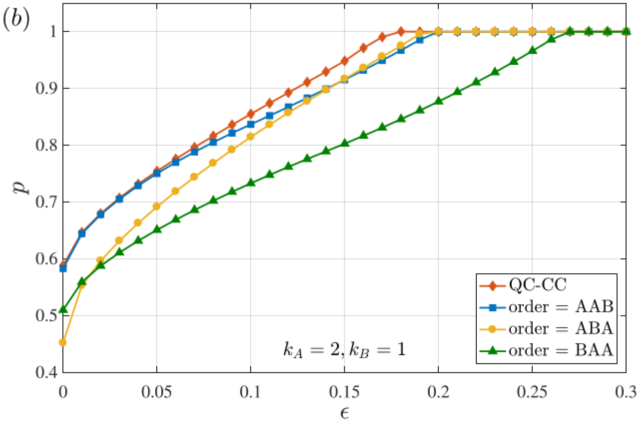


(k_A, k_B)	order	probability
$(1,1)$	AB	$p < \frac{4001}{10000}$
$(2,1)$	AAB	$p < \frac{5715}{10000}$
	ABA	$p < \frac{4919}{10000}$
	BAA	$p < \frac{5001}{10000}$

Quantum switch circuit simulation

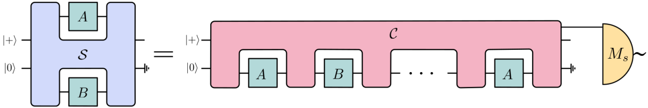
- The result is also robust, $F(\mathcal{S}, \mathcal{S}_{\text{sim}}) = 1 - \epsilon$





Quantum switch circuit simulation

- How about the probabilities when $k = 4$?



Quantum switch circuit simulation

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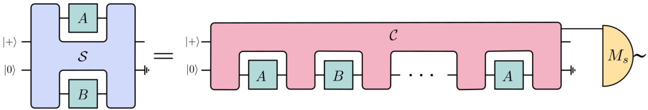
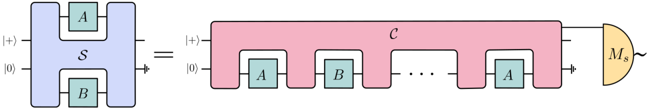


(2, 2)	AABB	$p < \frac{8307}{10000}$
	ABAB	$p < \frac{8484}{10000}$
	ABBA	$p < \frac{8695}{10000}$
(3, 1)	AAAB	$p < \frac{8373}{10000}$
	AABA	$p < \frac{6909}{10000}$
	ABAA	$p < \frac{7597}{10000}$
	BAAA	$p < \frac{6845}{10000}$

Quantum switch circuit simulation

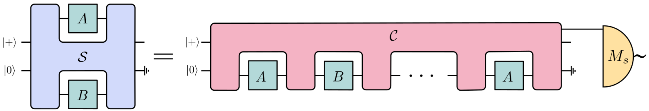
- How about identical channels?



k	order	probability
2	AA	$p < \frac{4001}{10000}$
3	AAA	$p < \frac{6534}{10000}$
4	AAAA	$p = 1$ (*)

Quantum switch circuit simulation

- How about identical channels?



k	order	probability
2	AA	$p < \frac{4001}{10000}$
3	AAA	$p < \frac{6534}{10000}$
4	AAAA	$p = 1$ (*)

If the target is not discarded, $p < 0.942$ for the AAAA order

Quantum switch circuit simulation

- How about unitary channels?

(k_A, k_B)	order (unitary only)	probability
$(1,1)$	AB	$p \approx 0.400$
$(2,1)$	AAB	$p \approx 0.596$
	ABA	$\boldsymbol{p = 1}$
	BAA	$p \approx 0.607$
$(2,2)$	AABB	$p = 1$ (*)
	ABAB	$\boldsymbol{p = 1}$
	ABBA	$\boldsymbol{p = 1}$
$(3,1)$	AAAB	$p \approx 0.708$
	AABA	$\boldsymbol{p = 1}$
	ABAA	$\boldsymbol{p = 1}$
	BAAA	$p = 1$ (*)

Quantum switch circuit simulation

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	AABA	$\boldsymbol{p = 1}$
	ABAA	$\boldsymbol{p = 1}$
	BAAA	$p = 1$ (*)

If the target is not discarded, $p < 0.822$ for the AABB order and $p < 0.667$ for the BAAA order

Quantum switch circuit simulation

- How these results were obtained?

Quantum switch circuit simulation

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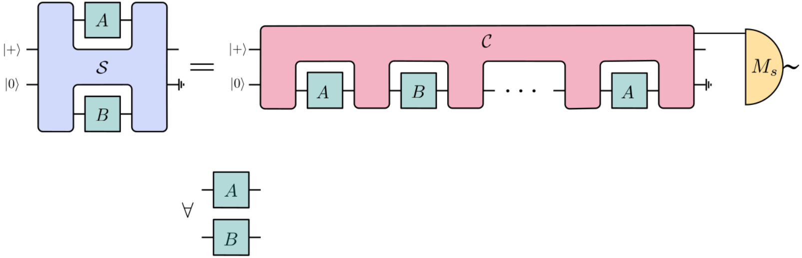
- Optimise over all inputs:



Quantum switch circuit simulation

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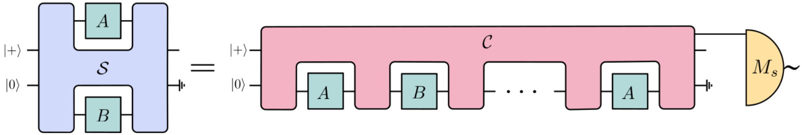

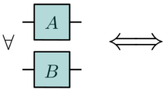

$\forall$ $\{A_i\}_i$ that forms a basis for the space spanned by k_A copies of a channel

$\{B_j\}_j$ that forms a basis for the space spanned by k_B copies of a channel

Quantum switch circuit simulation

- SDP (using splitting conic solver)

$$
\begin{aligned}
\textbf{given} \quad & \{J_i^A\}_i, \{J_j^B\}_j, k_A, k_B \\
\textbf{max} \quad & p \\
\textbf{s.t.} \quad & C_s * [(J_i^A)^{\otimes k_A} \otimes (J_j^B)^{\otimes k_B}] = p\, S * (J_i^A \otimes J_j^B) \ \ \forall i,j \\
& C_s \geq 0, \quad C - C_s \geq 0, \\
& \mathbb{P}(C) = C, \quad \mathrm{tr}(C) = d_{c_I} d_{t_I} d_{A_O}^{k_A} d_{B_O}^{k_B},
\end{aligned}
$$

$$
\begin{aligned}
\textbf{given} \quad & \{J_i^A\}_i, \{J_j^B\}_j, k_A, k_B \\
\textbf{min} \quad & \frac{1}{d_{A_I}^{k_A} d_{B_I}^{k_B} d_{c_O} d_{t_O}} \mathrm{tr}(\Gamma) \\
\textbf{s.t.} \quad & \sum_{i,j} \mathrm{tr}\big[R_{ij}\, (S * (J_i^A \otimes J_j^B))\big] = 1 \\
& \Gamma - \sum_{i,j} R_{ij} \otimes [(J_i^A)^{\otimes k_A} \otimes (J_j^B)^{\otimes k_B}]^T \geq 0 \\
& \Gamma \geq 0, \quad \overline{\mathbb{P}}(\Gamma) = \Gamma,
\end{aligned}
$$

any feasible point that yields some $p < 1$ constitutes a valid upper bound

Quantum switch circuit simulation

- But. . . is that a mathematical proof?

Quantum switch circuit simulation

- But. . . is that a mathematical proof?
- No! But, we can extract a proof out of it!

Quantum switch circuit simulation

- But... is that a mathematical proof?
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Algorithm:

1. Construct symbolic non-floating point operators Γ^{sym} and R^{sym}_{ij} by truncating them and obtaining a symbolic operator with only rational numbers.

2. Force the operators Γ^{sym} and R^{sym}_{ij} to be self-adjoint by making use of the expression $(M+M^\dagger)/2$, which is self-adjoint for any M.

3. Evaluate $t^{\text{sym}} := \sum_{i,j} \text{tr}\left[R^{\text{sym}}_{ij}(S * (J^A_i \otimes J^B_j))\right]$, where S, J^A_i, and J^B_j are also symbolic operators. Define $R^{\text{ok}}_{ij} := R^{\text{sym}}_{ij}/t^{\text{sym}}$ for all i,j.

4. Project Γ^{sym} onto the appropriate subspace to obtain $\overline{\mathbb{P}}(\Gamma^{\text{sym}})$.

5. Find $\eta \in \mathbb{R}$ such that $\Gamma^{\text{ok}} := \overline{\mathbb{P}}(\Gamma^{\text{sym}}) + \eta\mathbb{1} \geq 0$ and $\Gamma^{\text{ok}} - \sum_{i,j} R^{\text{ok}}_{ij} \otimes (J_i^{A^{\otimes k_A}} \otimes J_j^{B^{\otimes k_B}})^T \geq 0$

6. Output the quantity $\text{tr}(\Gamma^{\text{ok}})/d_{c_I} d_{t_I} d_{A_O}^{k_A} d_{B_O}^{k_B}$, which is a rigorous upper bound of the primal problem.

Quantum switch circuit simulation

- Very nice. . . But with the computer we are limited to a few queries. . .

Quantum switch circuit simulation

- Very nice... But with the computer we are limited to a few queries...
- Thm2: Let $d = 2^n$ be the dimension of the target state.
  If $k_A = 1$ and $k_B < 2^n$, there is no quantum circuit simulation of the quantum switch.

Quantum switch circuit simulation

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- Proof idea: We analyse the constraints obtained from imposing that the simulation holds for uniform convex combinations of unitary operators.

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  We then analyse the case of Pauli operations using a differentiation technique (from Analytical lower bound on query complexity for transformations of unknown unitary operations, T. Odake, S. Yoshida, M. Murao).

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  This allows us to note that imposing the simulation to hold has very strong implications.
  In particular, its eigendecomposition cannot be compatible with QCQC processes.

Quantum switch circuit simulation

- How to go beyond?

Quantum switch circuit simulation

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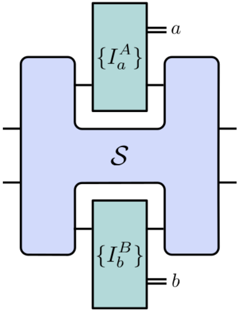
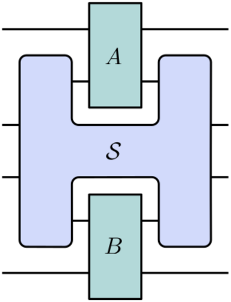
- The quantum switch is not restricted to single-partite channels.

Quantum switch circuit simulation

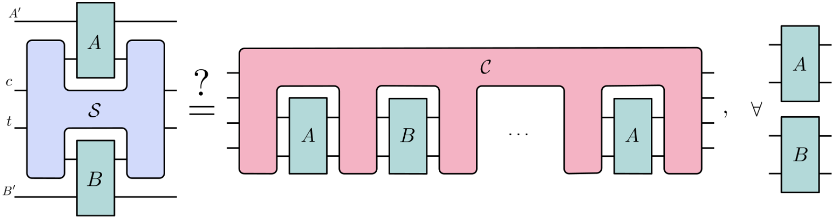
- How to go beyond?
- The quantum switch is not restricted to single-partite channels.
- How about instruments, bipartite channels?





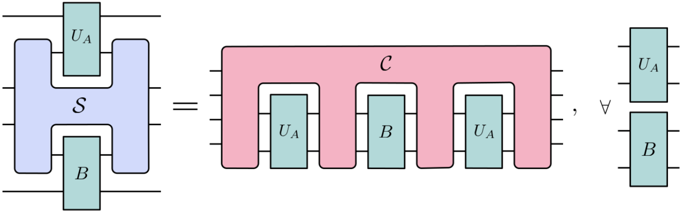
Quantum switch circuit simulation

General simulation:



Quantum switch circuit simulation

- Thm3:



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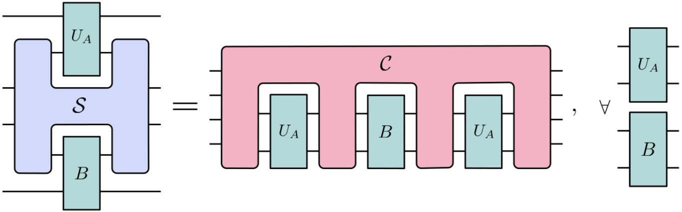
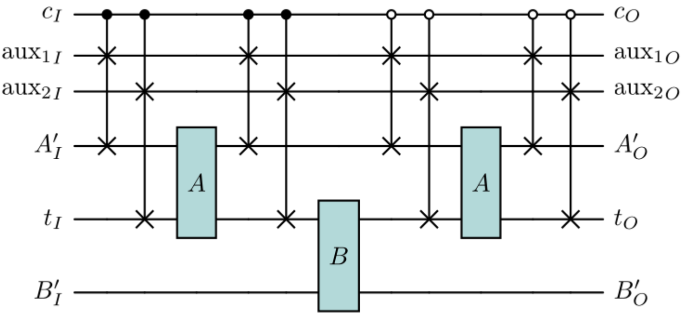


- The circuit:



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- Dream: "Simple task" that is easy if we have the quantum switch, but very hard if we only have standard circuits

Thank you