Parallel, sequential, and non-causal strategies for transforming unitary operations and discriminating quantum channel via a higher-order approach

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March 16, 2023

Quantum state transformations

$$|\psi_{\text{in}}\rangle \mapsto |\psi_{\text{out}}\rangle$$

Quantum state transformations

$$|\psi_{\text{in}}\rangle \mapsto |\psi_{\text{out}}\rangle = U|\psi_{\text{in}}\rangle$$

$$U^\dagger U = I$$

Quantum state transformations

$$\rho_{\text{in}} \mapsto \rho_{\text{out}}$$

Quantum state transformations

$$\rho_{\text{in}} \mapsto \rho_{\text{out}} = \widetilde{\Lambda}(\rho_{\text{in}})$$

$\widetilde{\Lambda}$ is CPTP

Quantum operation transformations

Can we transform quantum operations??

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$$U_{\text{in}} \mapsto U_{\text{out}}$$

$$\widetilde{\Lambda\text{in}} \mapsto \widetilde{\Lambda_{\text{out}}}$$

"Quantum" unitary inversion

$$U_d \mapsto U_d^{-1}$$

The universal/unknown paradigm

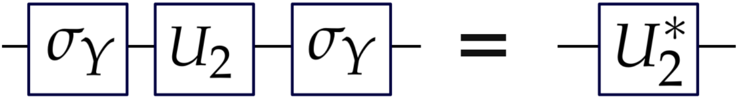
$$\sigma_Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \sigma_Z^{-1}$$

What do we want?

What do we want?

Ideally. . .

Something like this:



Phys. Rev. Research (2019)
J. Miyazaki, A. Soeda, and M. Murao

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  G. Chiribella and D. Ebler, New Journal of Physics (2016)

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- Universal (also works for “unknown” d-dimensional unitary)
- Exact
- Possible?
- Optimal average fidelity: $F_{max} = \frac{2}{d^2}$
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- $F_{max} < 1 \implies$ Impossible...

What do we want?

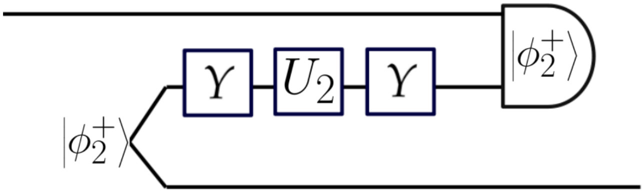
Probabilistic heralded?

What do we want?

Probabilistic heralded?

For qubits, Possible!

Explicit construction



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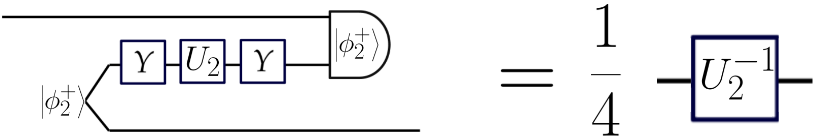
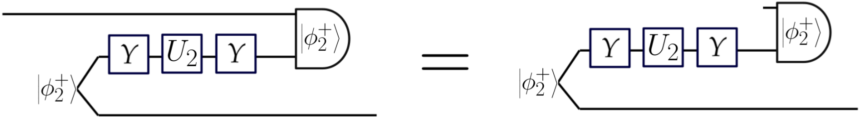


Delayed input state protocols



We want more!

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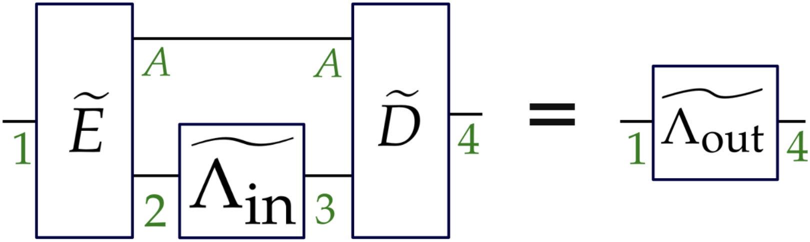
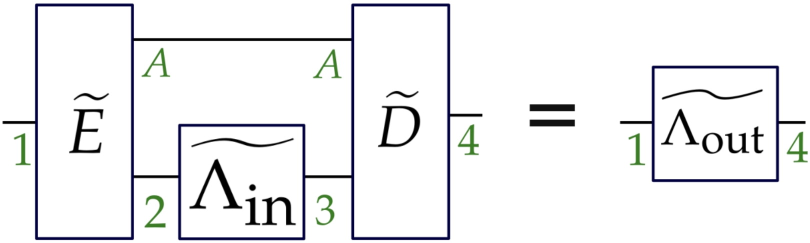
- Higher-order operations and supermaps!

Superchannels



The most general quantum superchannel?

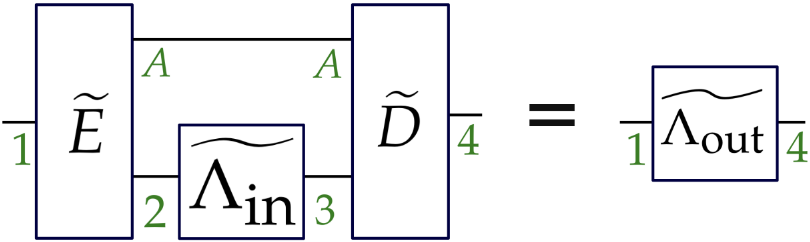
Superchannels



$$\widetilde{\widetilde{S}} : [\mathcal{L}(\mathcal{H}_2) \to \mathcal{L}(\mathcal{H}_3)] \to [\mathcal{L}(\mathcal{H}_1) \to \mathcal{L}(\mathcal{H}_4)]$$

- $\widetilde{\widetilde{S}}$ is a *linear supermap*

Superchannels



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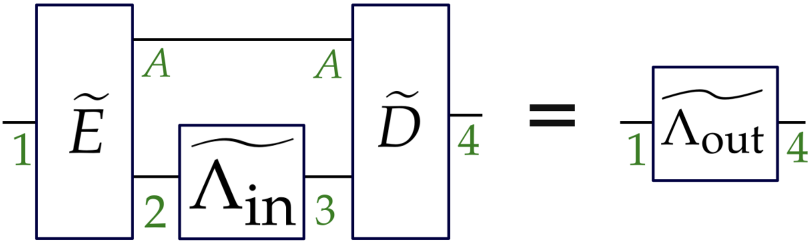

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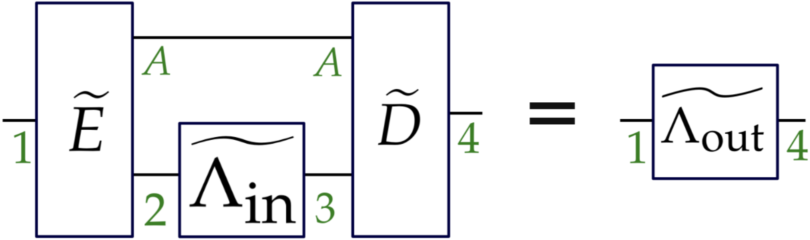
- $\tilde{\tilde{\tilde{S}}}$ may be applied into part of channel (completely CP preserving)

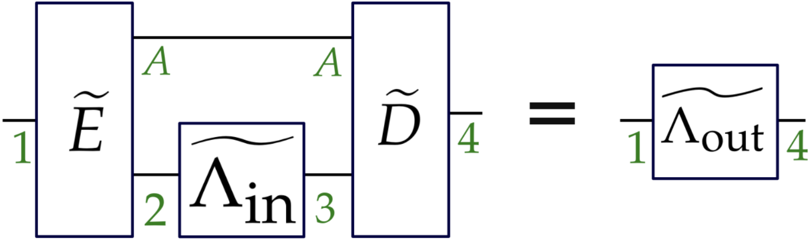
Superchannels



$$\widetilde{\widetilde{S}}(\widetilde{\Lambda_{in}}) = \mathrm{tr}_A \left(\widetilde{D} \circ \left(\widetilde{\Lambda_{in}} \otimes \widetilde{I}_A \right) \circ \widetilde{E} \right)$$

G. Chiribella, G. M. D'Ariano, and P. Perinotti EPL (2008)
K. Życzkowski J. Phys. A 41, 355302-23 (2008)
G. Gutoski and J. Watrous Proceedings of STOC (2007)

Superchannels



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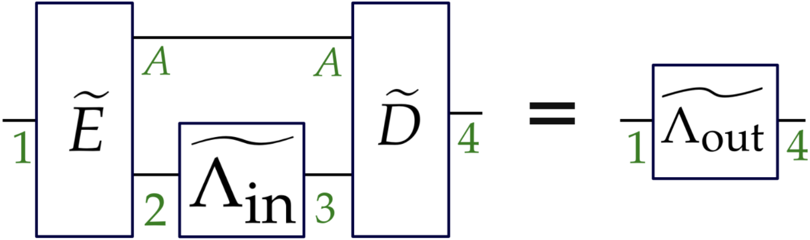

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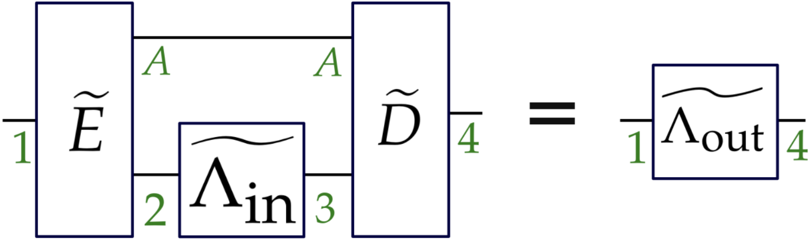

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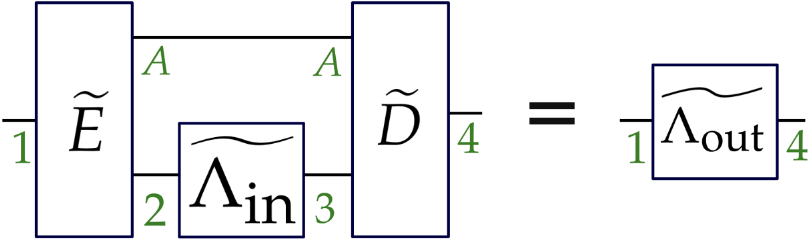

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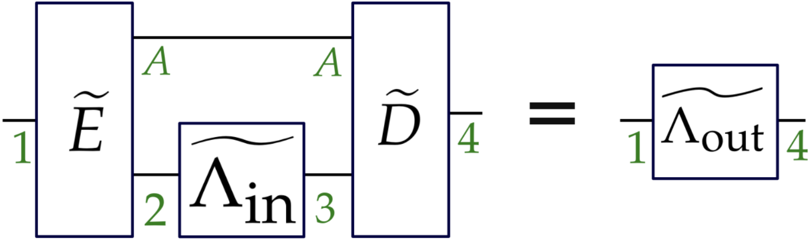

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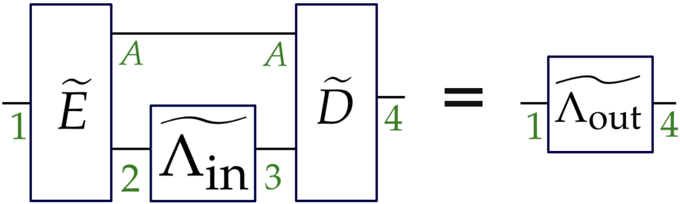
Superchannels



$S \in \mathcal{L}(\mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3 \otimes H_4)$ is a superchannel iff

$$S \geq 0$$

$$\text{tr}_4(S) = \text{tr}_{34}(S) \otimes \frac{I_4}{d_4}$$

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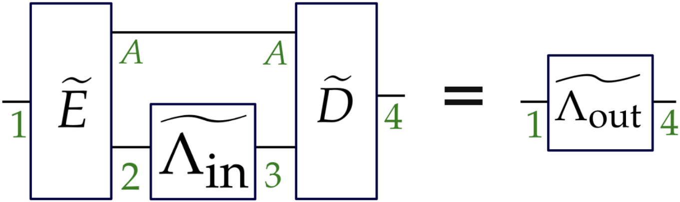
Superchannels



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Affine and positive semidefinite constraints $\implies$ SDP!!

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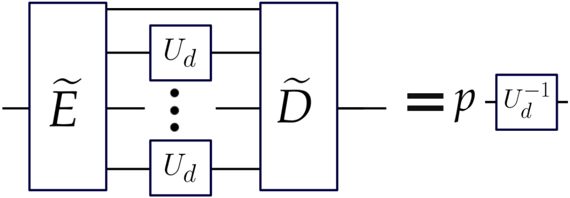

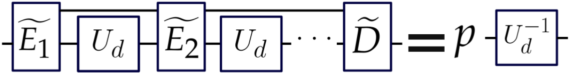

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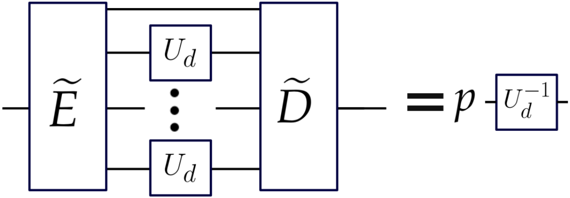
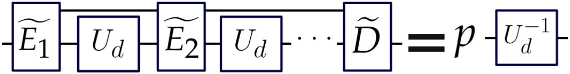
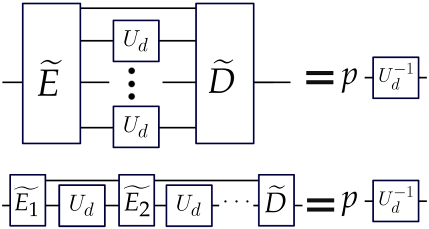




(Quantum combs, channel with memory, quantum strategy, quantum superchannels with multiple inputs)

Results



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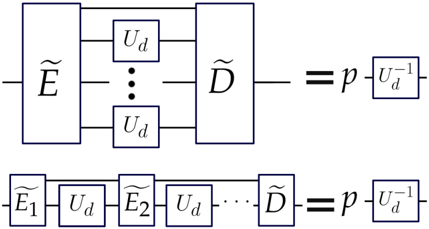

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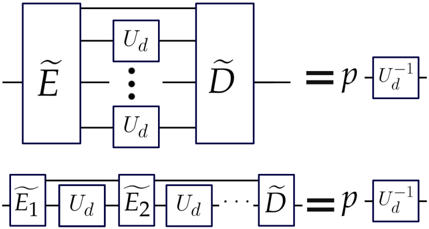

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$$1 - \frac{1}{k} \sim 1 - \frac{d^2-1}{\lfloor \frac{k}{d-1} \rfloor + d^2 - 1} \leq p \leq 1 - \frac{d^2-1}{k(d-1)+d^2-1} \sim 1 - \frac{1}{k}$$

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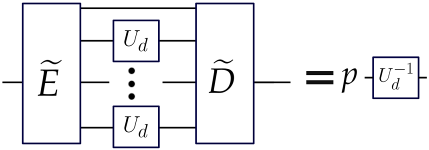

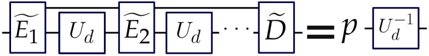

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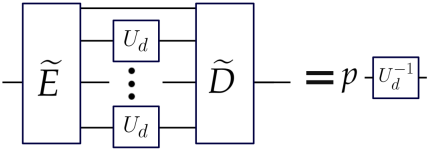

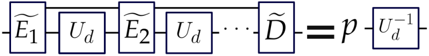

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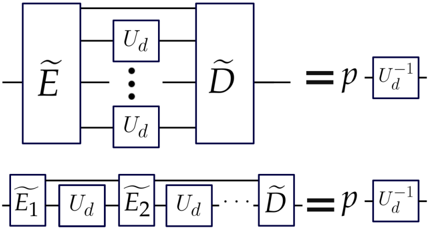

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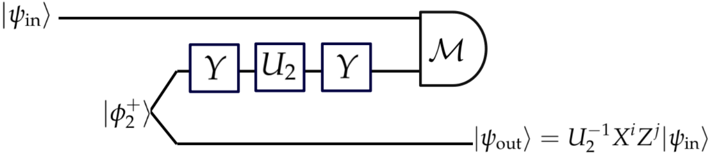
  MTQ, Q. Dong, A. Shimbo, A. Soeda, M. Murao
  PRA (2019), PRL (2019)

Qubit adaptive circuit



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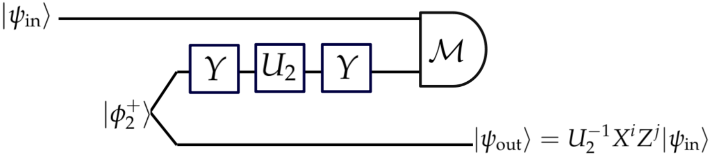
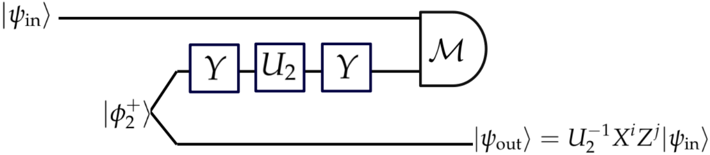


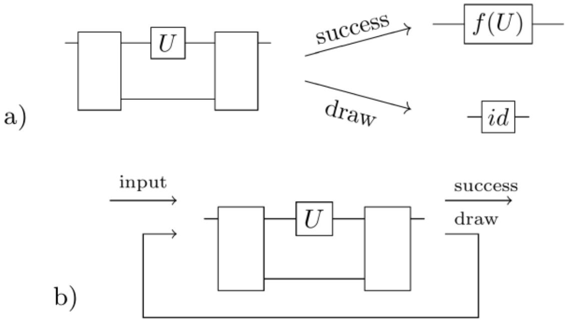
If we fail, we "destroy" the unknown input state...
then we cannot re-iterate this protocol...

Qubit adaptive circuit



But well...
if we can use U_2 once more:
apply $X^{-i} Z^{-j} U_2$ to recover the input state!

Success or draw



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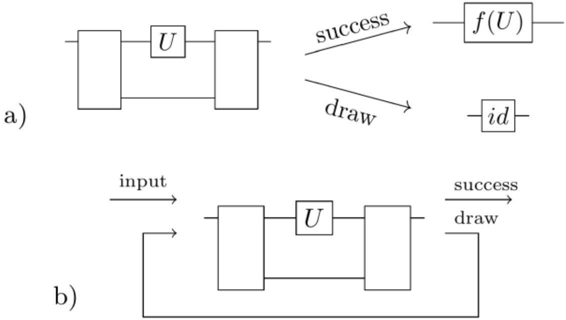


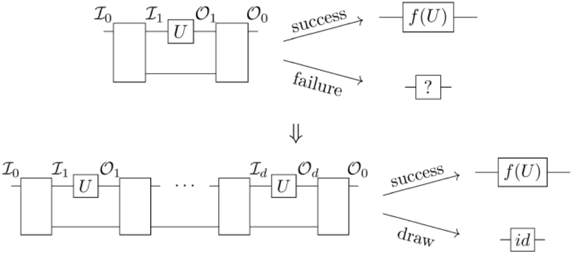
With k uses, this approach leads to a success probability of

$$p_s = 1 - (1 - p_{\text{draw}})^k$$

Success or draw

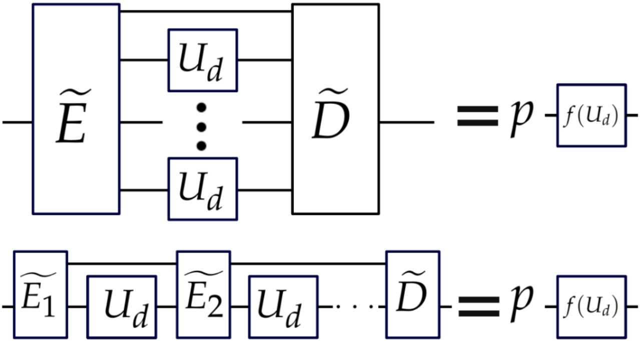
Theorem

Success or draw is always possible!



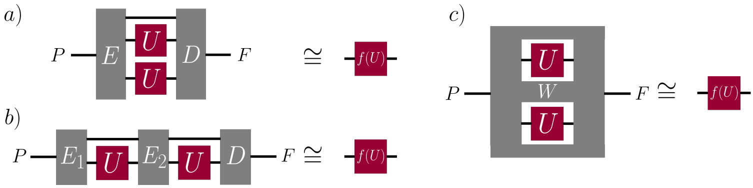
Q. Dong, **MTQ**, A. Soeda, M. Murao
PRL (2021)

Arbitrary functions $f(U_d)$



Deterministic protocols?

- Deterministic non-exact is also interesting!



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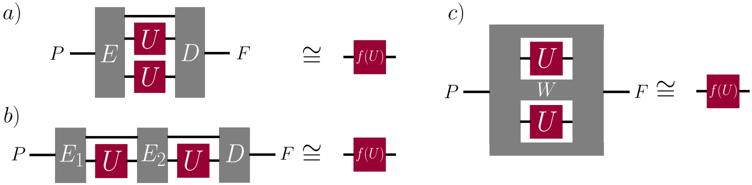

- Average channel fidelity (over the Haar measure)

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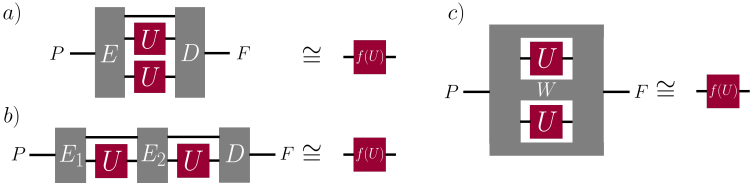

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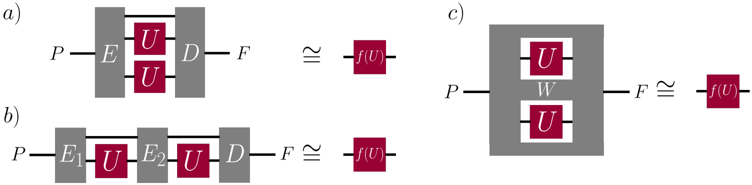
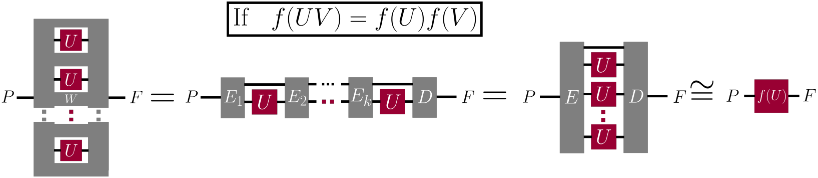


- Average channel fidelity (over the Haar measure)
- Maximal white noise visibility
- Parallel inversion = Parallel transposition = Unitary estimation

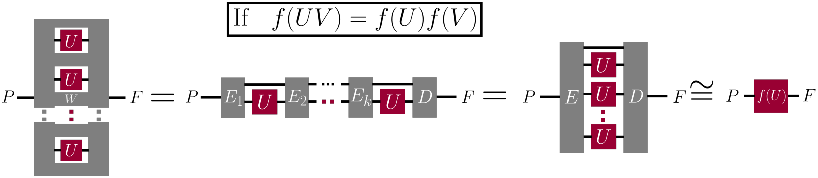
MTQ, D. Ebler, Quantum 2022

Solving the homomorphic case

- 

A. Bisio, G.M. D'Ariano, P. Perinotti, M. Sedlak
Physics Letters A (2014)

Solving the homomorphic case

- 

  A. Bisio, G.M. D'Ariano, P. Perinotti, M. Sedlak
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- *e.g.*, $(UV)^* = U^* V^*$, and cloning

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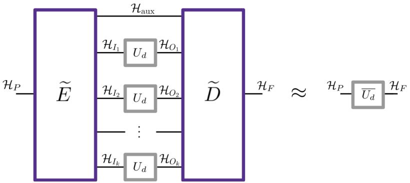

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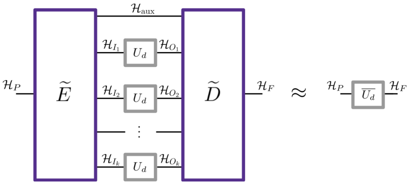

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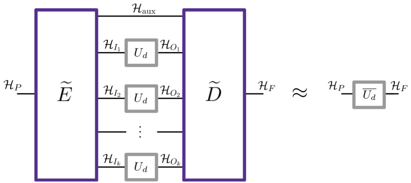

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arXiv:2206.00107
D. Ebler, M. Horodecki, M. Marciniak, T. Młynik, **MTQ**, M. Studziński

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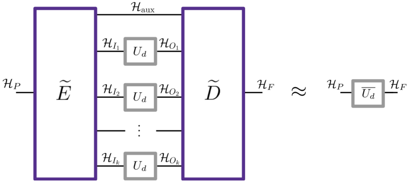

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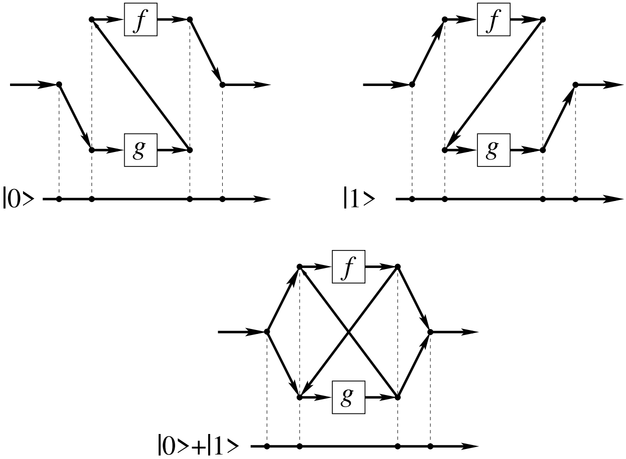
- Step 4: prove that if $k < d - 1$, $p = 0$

More general superchannels?

Can we go beyond sequential quantum circuits?

More general superchannels?

Quantum Switch:



Quantum computations without definite causal structure
G. Chiribella, G. M. D'Ariano, P. Perinotti, B. Valiron
PRA 2013

More general superchannels?

$$\widetilde{\widetilde{S}}(\widetilde{\Lambda_1} \otimes \widetilde{\Lambda_2}) = \widetilde{\Lambda_{out}}$$

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Process Matrices! (May have an indefinite causal order)

G. Chiribella, G. M. D'Ariano, P. Perinotti, and B. Valiron, PRA (2013)

O. Oreshkov, F. Costa, and Č. Brukner, Nature Communications (2012)

Process matrices

- Process matrices with indefinite causal order are not (explicitly) forbidden by quantum mechanics

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Summary of results

Deterministic unitary inversion: $U^{\otimes k} \overset{\approx}{\longmapsto} U^{-1}$

	Parallel	Sequential	General
$k=1$	$F=\frac{2}{d^2}$	$F=\frac{2}{d^2}$	$F=\frac{2}{d^2}$
$d=2$	$F=1-\sin^2\left(\frac{\pi}{k+3}\right)$	?	?
(d,k)	$\lVert M_{\text{est}} \rVert$	?	?
$k \to \infty$	$F \approx 1-\frac{1}{k^2}$	$1-\frac{1}{e^k} \leq F \leq ?$	?
Estimation	$F_{\text{par}} = F_{\text{est}}$	N/A	N/A
PBT	?	N/A	N/A
$k \leq d-1$	$F=\frac{k+1}{d^2}$	$F=\frac{k+1}{d^2}$	$F=\frac{k+1}{d^2}$
$d=2, k=4$	$F=1-\sin^2\left(\frac{\pi}{7}\right)$	$F=1$	$F=1$
$d=2, k=2$	$F=1-\sin^2\left(\frac{\pi}{5}\right)$	$F=\frac{3}{4}$	$F \approx 0.8249$
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Deterministic unitary transposition: $U^{\otimes k} \overset{\approx}{\longmapsto} U^{T}$

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$d=2$	$F=1-\sin^2\left(\frac{\pi}{k+3}\right)$	?	?
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Estimation	$F_{\text{par}}=F_{\text{est}}$	N/A	N/A
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Probabilistic unitary inversion: $U^{\otimes k} \mapsto p\,U^{-1}$

	Parallel	Sequential	General
$k=1$	$p=\frac{1}{d^2}$	$p=\frac{1}{d^2}$	$p=\frac{1}{d^2}$
$d=2$	$p=1-\frac{3}{k+3}$	?	?
(d,k)	?	?	?
$k\to\infty$	$p\approx 1-\frac{1}{k}$	$1-\frac{1}{e^k}\leq p\leq ?$	?
Store-retrieve	?	N/A	N/A
PBT	?	N/A	N/A
$k\leq d-1$	?	?	?
$d=2,k=4$	$p=\frac{4}{7}$	$p=1$	$p=1$
$d=2,k=2$	$p=\frac{2}{5}$	$p=\frac{3}{7}$	$p\approx 4/9$
$d=3,k=2$	$p\approx\frac{1}{9}$	$p\approx\frac{1}{9}$	$p\approx\frac{1}{9}$

Probabilistic unitary transposition: $U^{\otimes k} \mapsto p\,U^{T}$

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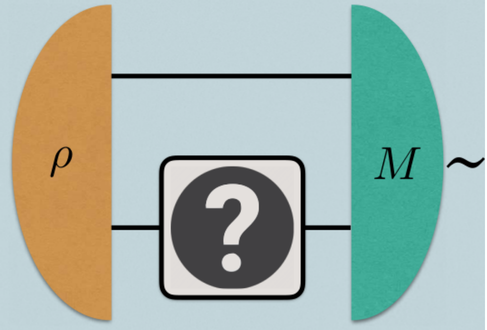
$k=1$	$p=\frac{1}{d^2}$	$p=\frac{1}{d^2}$	$p=\frac{1}{d^2}$
$d=2$	$p=1-\frac{3}{k+3}$	?	?
(d,k)	$p=1-\frac{d^2-1}{k+d^2-1}$	?	?
$k\to\infty$	$p\approx 1-\frac{1}{k}$	$1-\frac{1}{e^k}\leq p\leq ?$	?
Store-retrieve	$p_{\text{par}}=p_{\text{s-r}}$	N/A	N/A
PBT	$p_{\text{par}}=p_{\text{pbt}}$	N/A	N/A
$k\leq d-1$	$p=1-\frac{d^2-1}{k+d^2-1}$	?	?
$d=2,k=4$	$p=\frac{4}{7}$	$p=1$	$p=1$
$d=2,k=2$	$p=\frac{2}{5}$	$p=\frac{3}{7}$	$p\approx 4/9$
$d=3,k=2$	$p=\frac{2}{10}$	$p\approx\frac{2}{9}$	$p\approx\frac{2}{8}$

How can one perform a measurement on a quantum operation?

Measuring quantum operations



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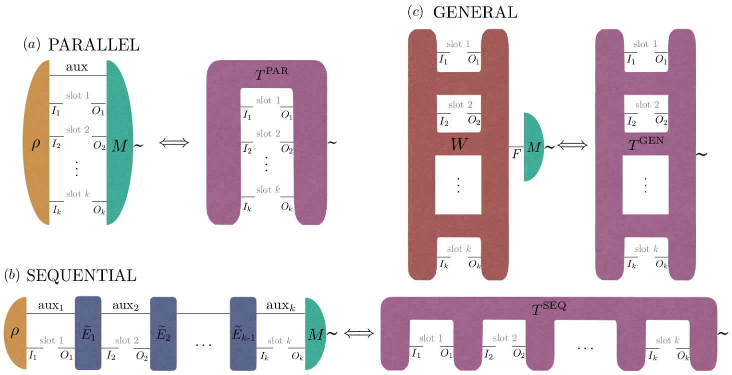

J. Bavaresco, M. Murao, MTQ PRL (2021)
J. Bavaresco, M. Murao, MTQ J. Math. Phys. (2022)

Quantum channel discrimination

- Two unitaries $\implies$ parallel is optimal

  G. Chiribella, G. M. D'Ariano, P. Perinotti PRL (2008)

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  J. Bavaresco, M. Murao, **MTQ**, J. Math. Phys. (2022)

- Non-symmetric scenarios? Non-unitary quantum channels?

- Almost always, we have a strict hierarchy!
  J. Bavaresco, M. Murao, **MTQ** PRL (2021)
  J. Bavaresco, M. Murao, **MTQ** J. Math. Phys. (2022)

Final remarks

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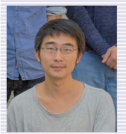
- Can we have deterministic exact unitary inversion with finite sequential circuits?
- Practical and realistic implementations

Thank you!

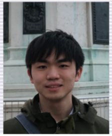
Mio Murao

Jisho Miyazaki

Atsushi Shimbo



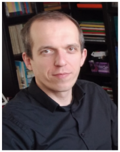
Qingxiuxiong Dong



Satoshi Yoshida

Akihito Soeda

Jessica Bavaresco



Michał Studziński

Michał Horodecki

Tomasz Młynik

Daniel Ebler

Thank you!