Success-or-draw: A strategy allowing repeat-until-success in quantum computation

Qingxiuxiong Dong, Marco Túlio Quintino, Akihito Soeda, Mio Murao

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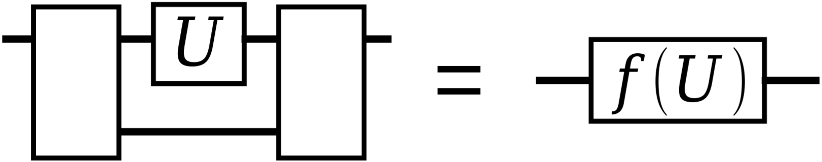
Phys. Rev. Lett. 126, 150504 (2021) – arXiv:2011.01055

Probabilistic quantum unitary transformations

$$U \mapsto f(U)$$

Probabilistic quantum unitary transformations



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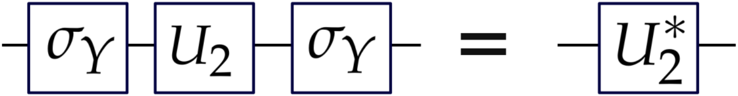

Phys. Rev. Research 1, 013007 (2019)
J. Miyazaki, A. Soeda, and M. Murao

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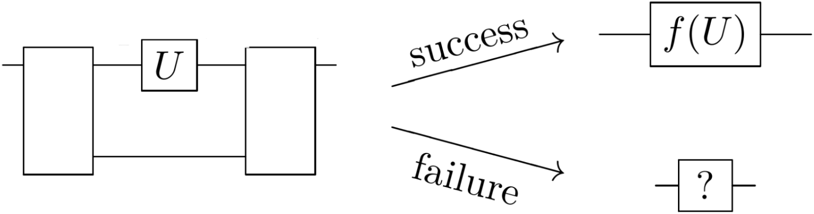

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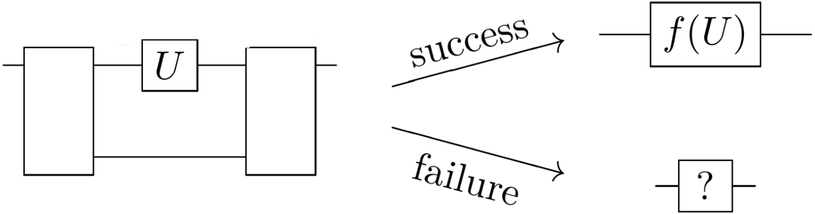

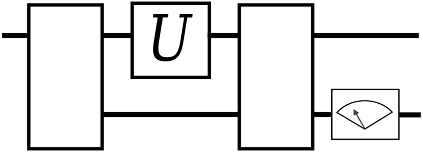

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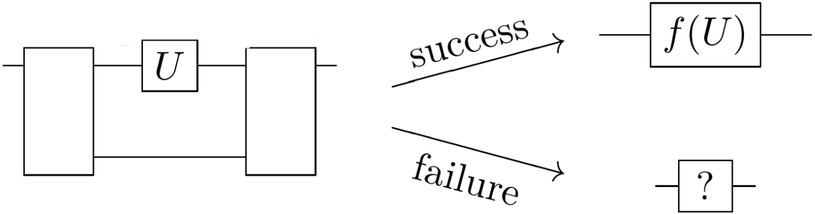
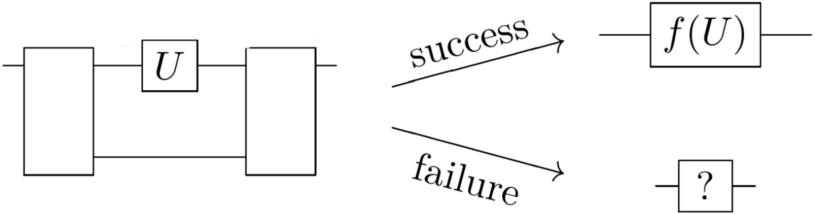


- Universal (also works for "unknown" d-dimensional unitary)

Probabilistic quantum unitary transformations



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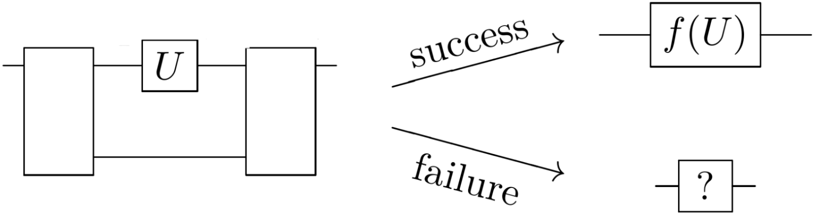
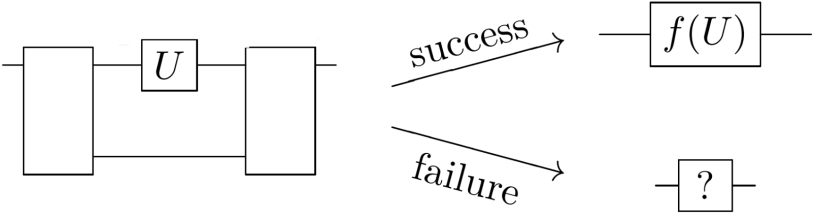


- Universal (also works for "unknown" d-dimensional unitary)
- Probabilistic but exact
- We know when it fails (Probabilistic heralded)

Increasing the success probability

The desired transformation is possible with probability p_U

$$U \mapsto p_U f(U)$$



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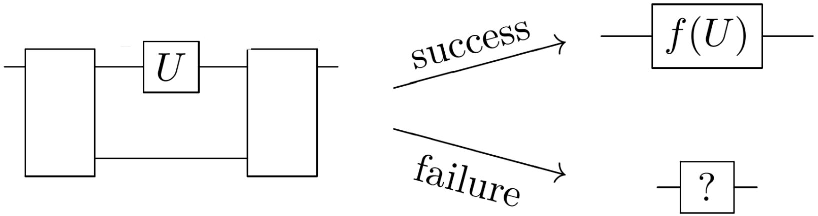

How can we increase the success probability?

Increasing the success probability

Standard trick: repeat-until-success

Probabilistic quantum unitary transformations

- Coin tossing:

Probabilistic quantum unitary transformations

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- If p is the probability of getting heads in a single toss

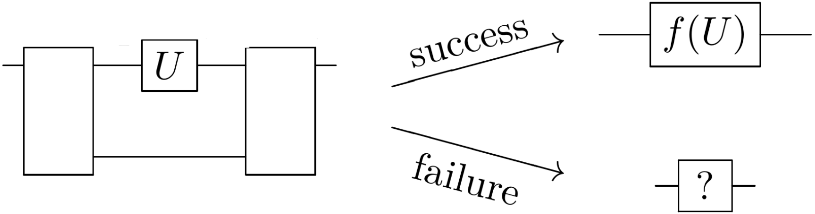
Probabilistic quantum unitary transformations

- Coin tossing:

- If p is the probability of getting heads in a single toss
- With n trials, the probability of getting heads at least once:

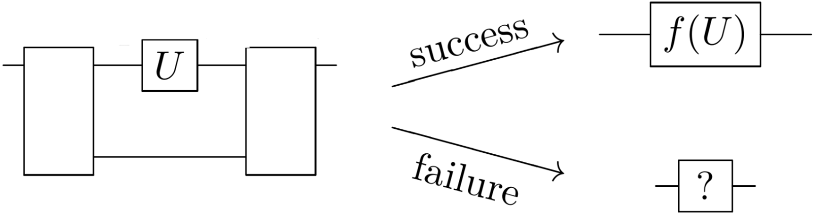
$$p_s(n) = 1 - (1-p)^n$$

Probabilistic quantum unitary transformations



- That's great, if $U \mapsto p_U f(U)$, repeat-until-success provides a method where the probability of failure decreases exponentially!

Probabilistic quantum unitary transformations



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- But. . .

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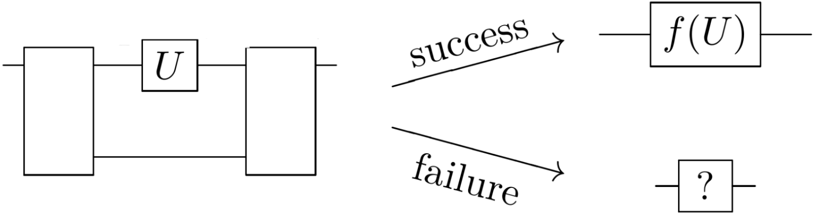

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- It depends on *how* we fail...

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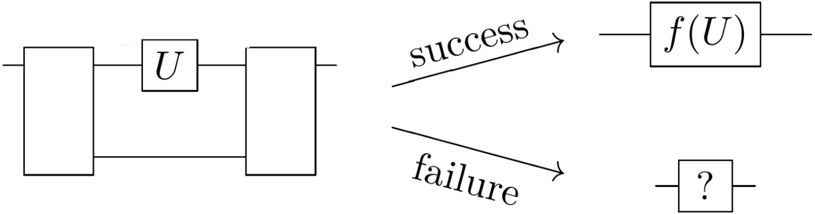
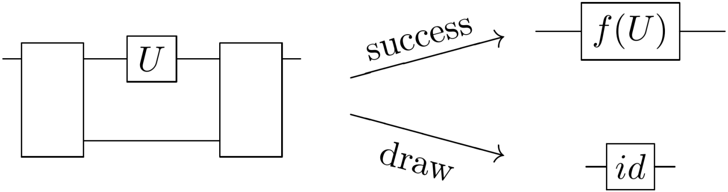


- That's great, if $U \mapsto p_U\, f(U)$, repeat-until-success provides a method where the probability of failure decreases exponentially!
- But...
- It depends on *how* we fail...
- We *need* success-or-draw!

Probabilistic quantum unitary transformations



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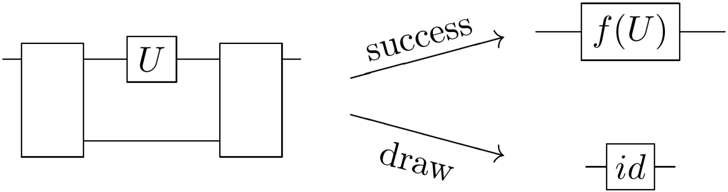

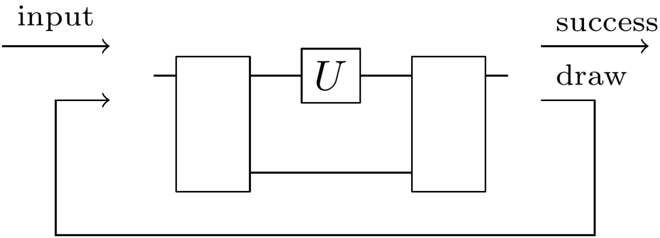

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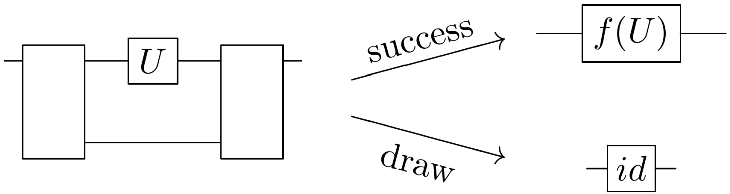
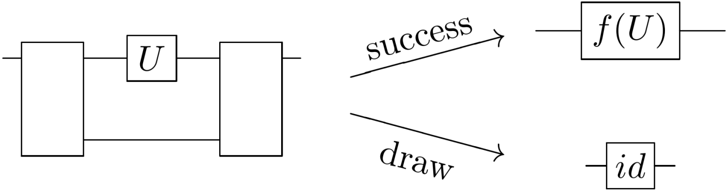


- But well. . . can such a thing exist?

Probabilistic quantum unitary transformations



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- What about "measurement disturbs the state", "no-cloning" and all these quantum bad guys?

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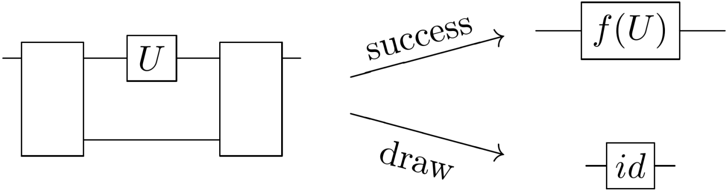

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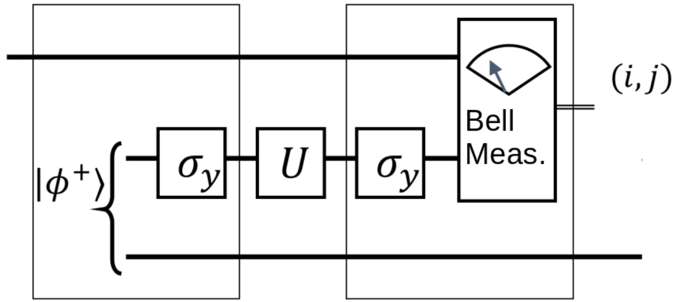
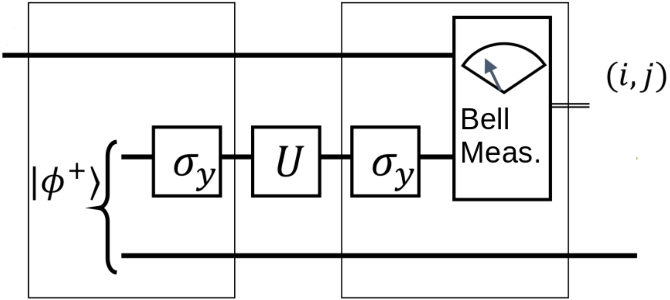
- Example?

Explicit construction



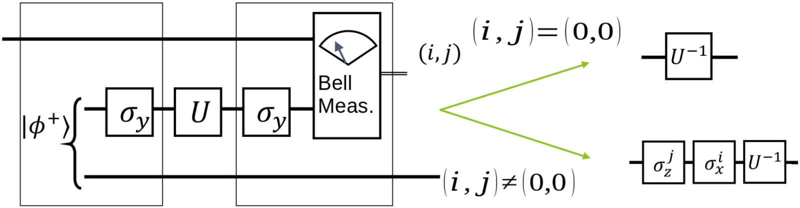
Reversing Unknown Quantum Transformations:
Universal Quantum Circuit for Inverting General Unitary Operations
M. T. Quintino, Q. Dong, A. Shimbo, A. Soeda, M. Murao
Phys. Rev. Lett. 123, 210502 (2019)

Explicit construction



$$|\phi^+\rangle := \frac{|00\rangle + |11\rangle}{\sqrt{2}}, \qquad \mathcal{M} := \left\{ id \otimes \left(\sigma_X^i \sigma_Z^j\right) |\phi^+\rangle \right\}_{ij}$$

Explicit construction



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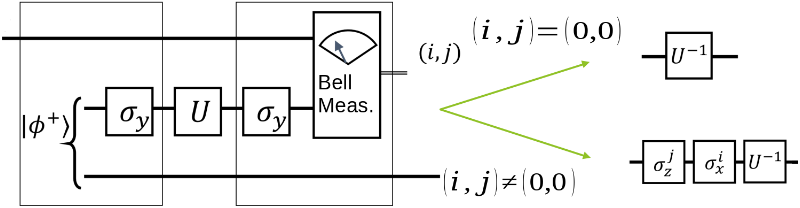
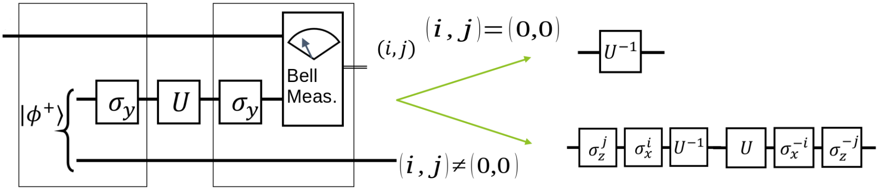


Since U is "unknown", we cannot invert $\sigma_Z^i \sigma_X^j U^{-1}$

Explicit construction



An extra call of U allows success-or-draw

Success-or-draw

- For qubit unitary inversion, with 2-calls, success-or-draw is possible!

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- Actually, success-or-draw is *always* possible!

Success-or-draw

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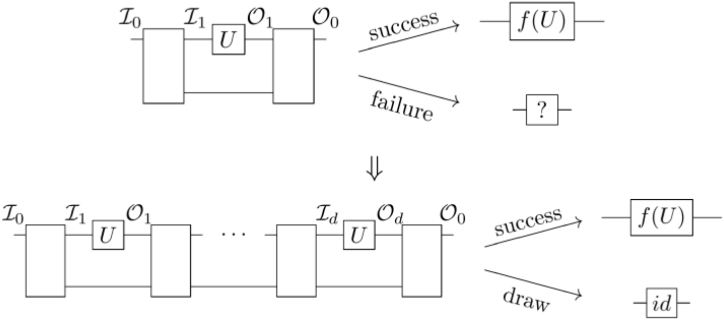
- But...
- qubit unitary inversion...
- what a particular example...
- how about general transformations $f(U)$??
- Actually, success-or-draw is *always* possible!
- and we need at most d calls of the input-gate U

Success-or-draw

Theorem

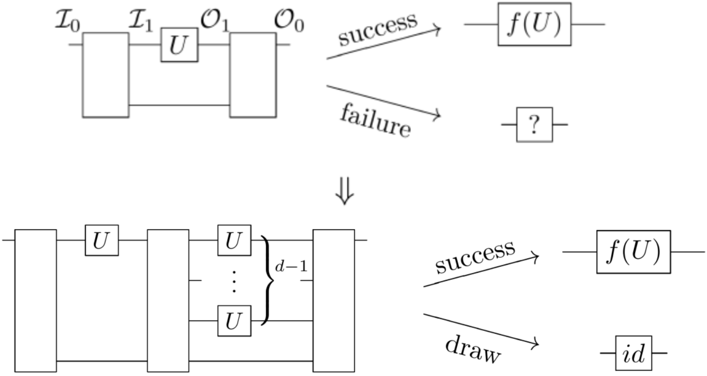
Success-or-draw is always possible!



Success-or-draw

Theorem

Success-or-draw is always possible with length 2 and d calls or U:



Success-or-draw

Theorem

If there exists a quantum circuit such that

$$U \mapsto p_U f(U)$$

there is a depth-2 success-or-draw circuit such that

$$U^{\otimes d} \mapsto \epsilon p_U f(U), \qquad \epsilon > 0$$

Methods

- Quantum combs, superinstruments, Choi representation
  Quantum Circuits Architecture
  G. Chiribella, G.M. D'Ariano, P. Perinotti
  Phys. Rev. Lett. 101, 060401 (2008)

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- We seek for $S \geq 0$ and $N \geq 0$ such that $S + N$ is a k-slot comb,
$$S * |U\rangle\rangle\langle\langle U|^{\otimes k} = \epsilon p_U |f(U)\rangle\rangle\langle\langle f(U)|$$
  and
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$$N * \Pi_{\mathrm{sym}}^{\mathcal{I}\mathcal{O}} \propto |id\rangle\rangle\langle\langle id|$$
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  where $\Pi_{\text{sym}} = \sum_\sigma P_\sigma^{\mathcal{I}} \otimes P_\sigma^{\mathcal{O}}$
- We set $S := \epsilon S_t \otimes id$ and construct a neutralising operator N via a nice Pauli decomposition

Success-or-draw

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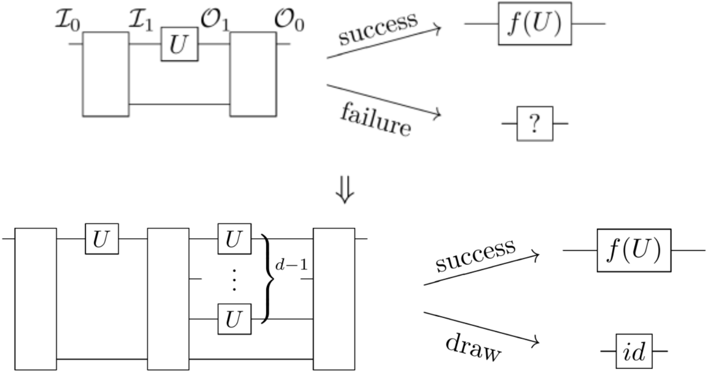


OK, but how about this ϵ?

Optimising the success probability

Given a desired function f and the number of uses k, maximising the success-or-draw circuit

$$U^{\otimes k} \mapsto pf(U)$$

is a semidefinite programming (SDP) problem!

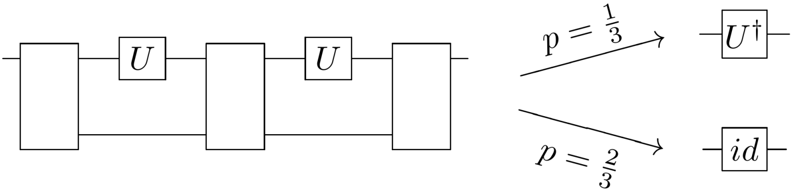
Success-or-draw

SDP:

$$\begin{aligned} &\max p \\ \text{such that: } & S * |U\rangle\rangle\langle\langle U|^{\otimes k} = p|f(U)\rangle\rangle\langle\langle f(U)|, \quad \forall U \\ & N * |U\rangle\rangle\langle\langle U|^{\otimes k} \propto |id\rangle\rangle\langle\langle id|, \quad \forall U \\ & S + N = C \text{ is a } k\text{-slot quantum comb} \end{aligned}$$

Success-or-draw

SDP allows us to find interesting things
Qubit unitary inversion can be improved:



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Summary

- Repeat-until-success strategies is a simple/natural method to ensure success
- For that, we need success-or-draw
- Success-or-draw is *always* possible
- We require d uses of the unitary in a length 2 circuit
- Particular cases may be analysed via SDP

Future?

- Proof of principle + fine analysis via SDP, but no “friendly” intermediate result

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- Proof of principle + fine analysis via SDP, but no “friendly” intermediate result
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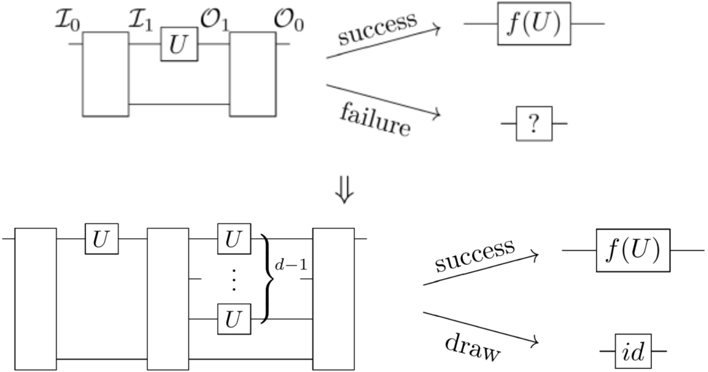
- It would be nice to see a more concrete application. . .

Thank you!



$\mathcal{I}_0$
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U
$\mathcal{O}_1$
$\mathcal{O}_0$
success
$f(U)$
failure
?
⇓
U
U
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draw
id