Adaptive circuits exponentially outperform parallel ones for universal unitary inversion

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Phys. Rev. Lett. 123, 180401 and Phys. Rev. A 100, 062339

"Quantum" unitary inversion

$$U_d \mapsto U_d^{-1}$$

The universal/unknown paradigm

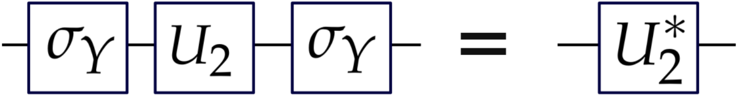
$$\sigma_Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \sigma_Z^{-1}$$

What do we want?

What do we want?

Ideally. . .

Something like this:



Phys. Rev. Research 1, 013007 (2019)
J. Miyazaki, A. Soeda, and M. Murao

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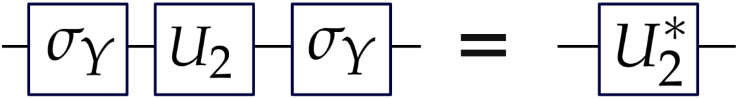

$$\sigma_Y U_2 \sigma_Y = U_2^*, \ \forall U_2 \in SU(2)$$

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What do we want?

Understand better transformations between operations

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- Universal (also works for “unknown” d-dimensional unitary)

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- Optimal average fidelity: $F_{max} = \frac{2}{d^2}$
  G. Chiribella and D. Ebler, New Journal of Physics (2016)

What do we want?

- Universal (also works for "unknown" d-dimensional unitary)
- Exact
- Possible?
- Optimal average fidelity: $F_{max} = \frac{2}{d^2}$
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- $F_{max} < 1 \implies$ Impossible...

What do we want?

Probabilistic heralded?

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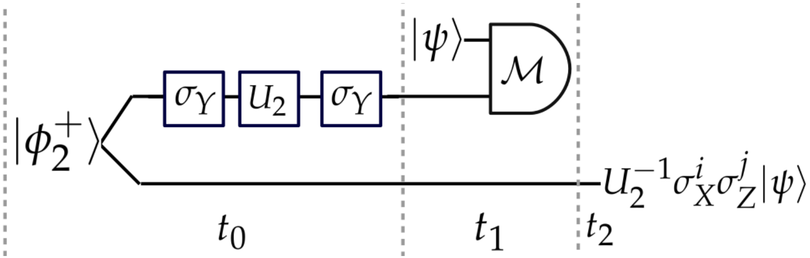
Probabilistic heralded?

For qubits, Possible!

Explicit construction



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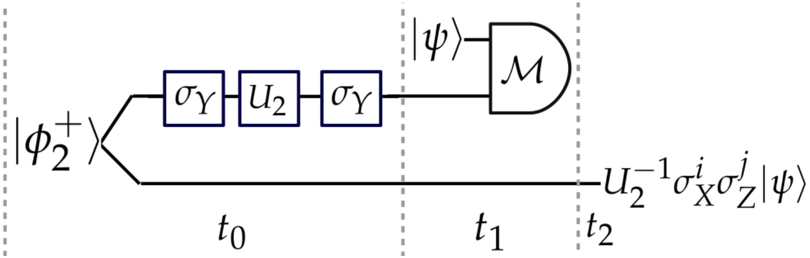

$$\text{With } p = \frac{1}{4},\ U_2 \mapsto U_2^{-1}$$

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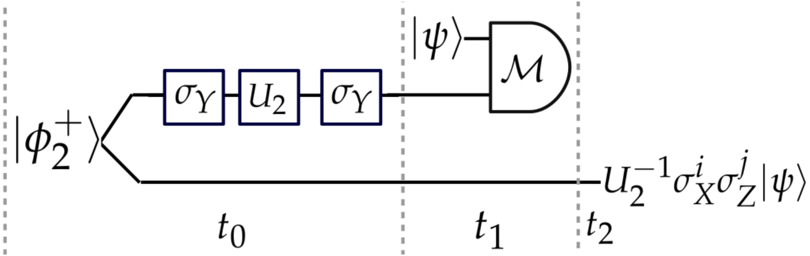

With $p = \frac{1}{4}$, $U_2 \mapsto U_2^{-1}$

Delayed input-state

We want more!

- Is it optimal?

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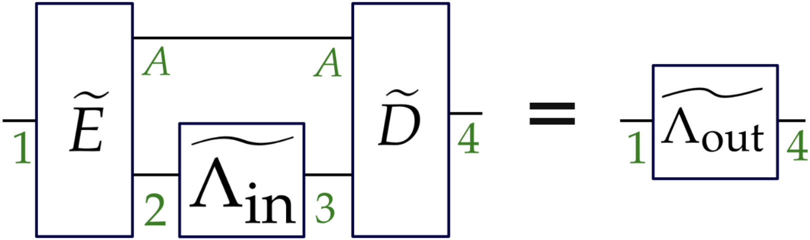
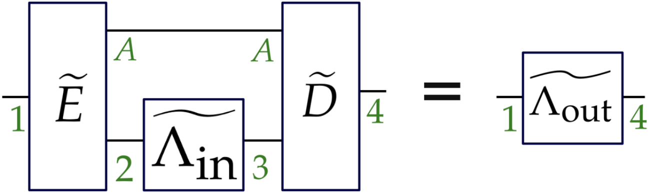
- Higher-order operations and supermaps!

Superchannels



G. Chiribella, G. M. D'Ariano, and P. Perinotti, EPL (2008)
K. Życzkowski J. Phys. A 41, 355302-23 (2008)

Superchannels



The most general quantum superchannel?

G. Chiribella, G. M. D'Ariano, and P. Perinotti, EPL (2008)
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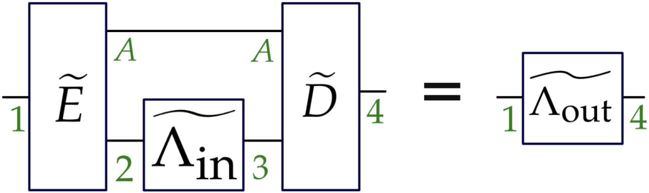

The most general quantum superchannel:

$$\widetilde{\widetilde{S}}(\widetilde{\Lambda_{\text{in}}}) = \widetilde{\Lambda_{\text{out}}} = \widetilde{D} \circ \left(\widetilde{\Lambda_{\text{in}}} \otimes \widetilde{I}_A\right) \circ \widetilde{E}$$

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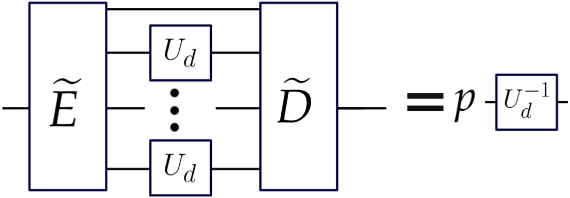
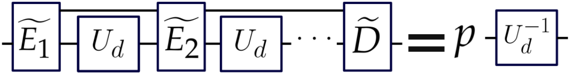
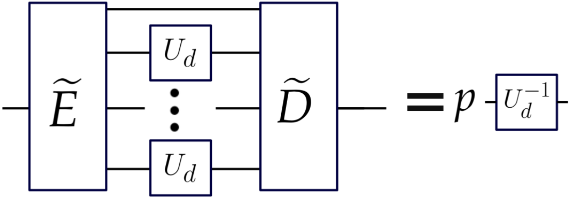
- How can we increase the success probability?
- More calls/copies

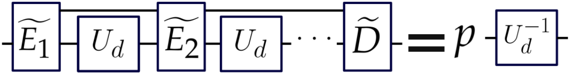




(Quantum combs, channel with memory, quantum strategy, quantum channels with sequential multiple uses)

Results





- Parallel ($d = 2$): $p = 1 - \frac{3}{k+3}$

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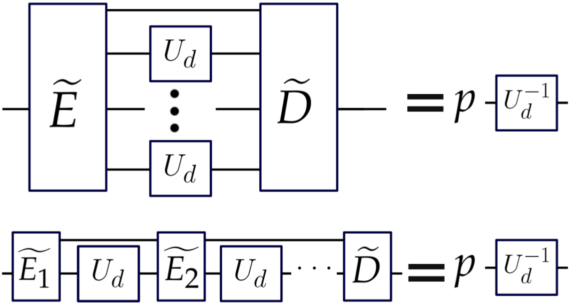

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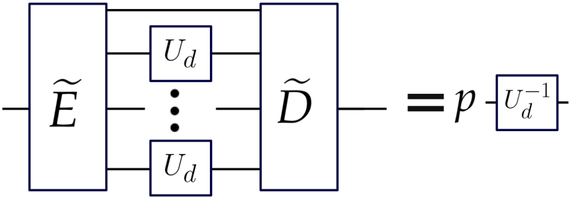

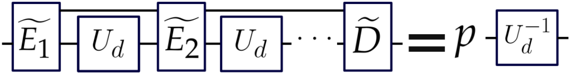

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$$1 - \frac{1}{k} \sim 1 - \frac{d^2-1}{\lfloor \frac{k}{d-1} \rfloor + d^2 - 1} \leq p \leq 1 - \frac{d^2-1}{k(d-1)+d^2-1} \sim 1 - \frac{1}{k}$$

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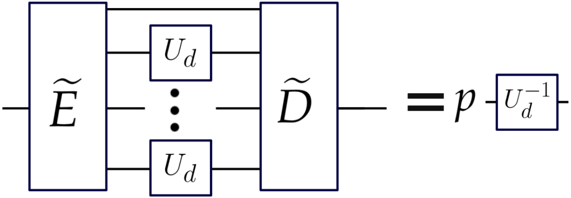

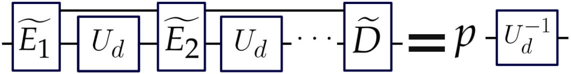

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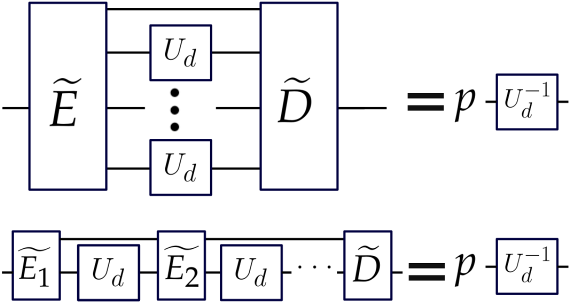

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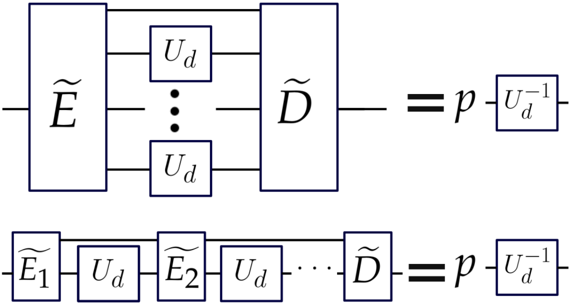

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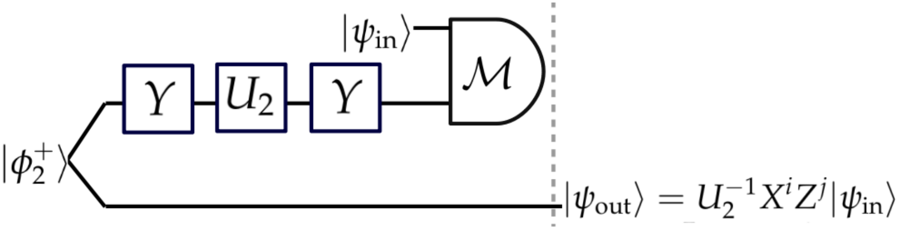
- Optimal parallel $\implies$ delayed input-state
- Sequential ($k < d - 1$): $p = 0$
- Sequential ($k \geq d - 1$): $p \geq 1 - \left(1 - \frac{1}{d^2}\right)^{\lceil \frac{k+2-d}{d} \rceil} \sim 1 - \frac{1}{e^k}$

Qubit adaptive circuit

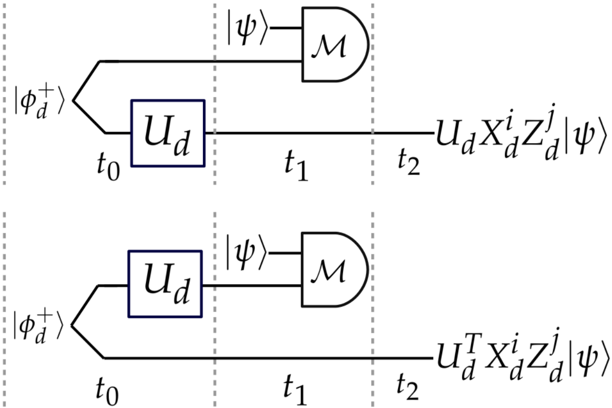


if $i = j = 0$, one has $|\psi_{\rm out}\rangle = U_2^{-1}|\psi_{\rm in}\rangle$

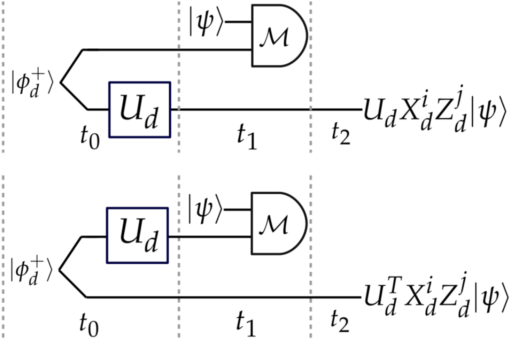
else, apply $Z^{-j} X^{-i} U_2$ on $|\psi_{\rm out}\rangle$,

recover $|\psi_{\rm in}\rangle$ and re-start the protocol

Unitary transposition

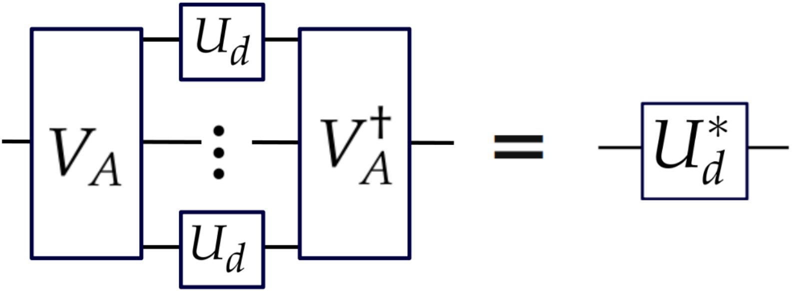


Unitary complex conjugation



$$\sigma_Y U_2 \sigma_Y = U_2^*$$

Unitary complex conjugation



$$V_A : \mathbb{C}_d \to \mathbb{C}_d^{\otimes d-1}$$

$$V_A := \sum_{\tau \in S_d} \frac{\mathrm{sgn}(\tau)}{\sqrt{(d-1)!}} |\tau_2, \ldots, \tau_d\rangle \langle \tau_1|$$

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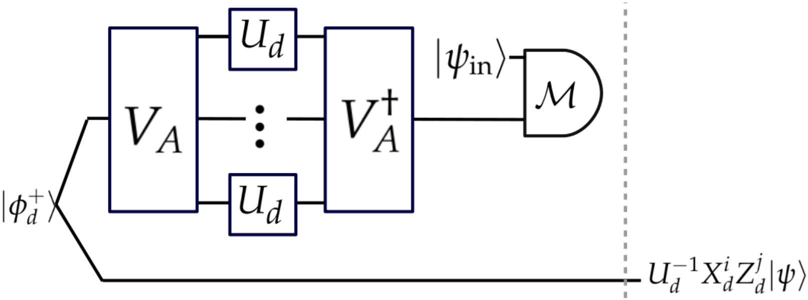
Qudit $k = d - 1$ parallel circuits



Optimal parallel unitary complex conjugation

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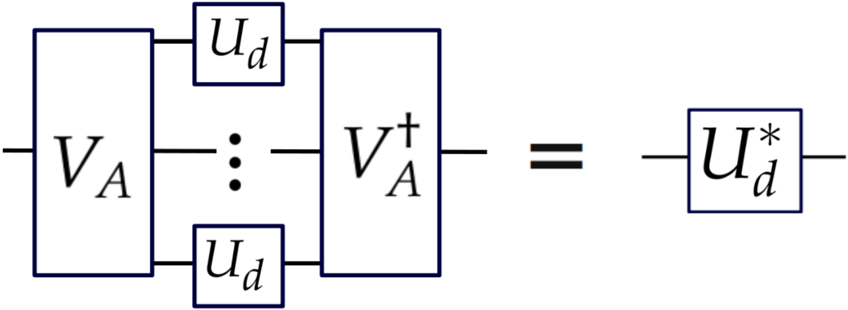
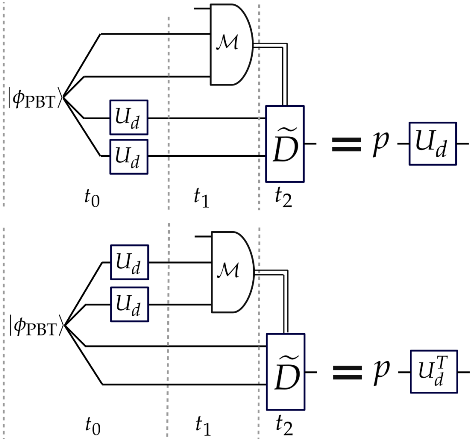


$$k < d - 1 \implies p = 0$$

Optimal parallel unitary transposition

Port-Based Teleportation: S. Ishizaka and T. Hiroshima, PRL (2008)
M. Studziński, S. Strelchuk, M. Mozrzymas, M. Horodecki, Sci. Rep. (2017)
Unitary store and retrieve: M. Sedlák, A. Bisio, and M. Ziman, PRL (2019):



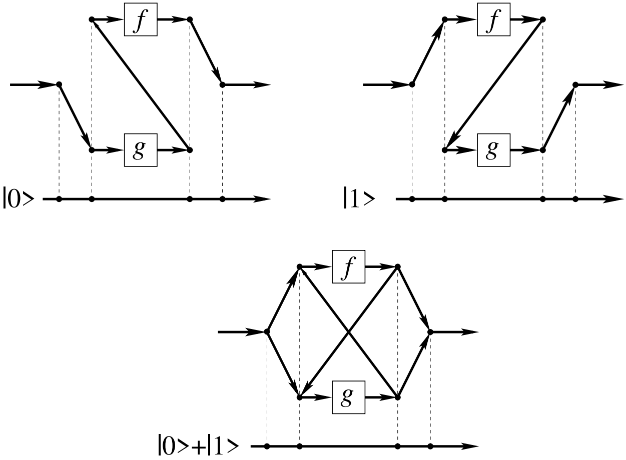
$$p = 1 - \frac{d^2-1}{k+d^2-1}$$

More general superchannels?

Can we go beyond sequential quantum circuits?

More general superchannels?

Quantum Switch:



Quantum computations without definite causal structure
G. Chiribella, G. M. D'Ariano, P. Perinotti, B. Valiron
PRA 2013

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$$\widetilde{\widetilde{S}}(\widetilde{\Lambda_1} \otimes \widetilde{\Lambda_2}) = \widetilde{\Lambda_{out}}$$

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Process Matrices! (May have an indefinite causal order)
G. Chiribella, G. M. D'Ariano, P. Perinotti, and B. Valiron, PRA (2013)
O. Oreshkov, F. Costa, and Č. Brukner, Nature Communications (2012)

Process matrices

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- If $k \geq d - 1$, $p =$???

SemiDefinite Programming

$$\max p$$
$$s.t.$$
$$\widetilde{\widetilde{S}}(\widetilde{U_d^{\otimes k}}) = p\widetilde{U_d^{-1}}, \qquad \forall \widetilde{U_d}$$
$$\widetilde{\widetilde{S}} \in \text{Some desired set}$$

Where, $\widetilde{U_d}(\rho) := U_d \rho U_d^{-1}$

Maximall success probability

$d=2$	Parallel	Sequential	Indefinite causal order
$k=1$	$\frac{1}{4}=0.25$	$\frac{1}{4}=0.25$	$\frac{1}{4}=0.25$
$k=2$	$\frac{2}{5}=0.4$	$0.4286\approx\frac{3}{7}$	$0.4444\approx\frac{4}{9}$
$k=3$	$\frac{1}{2}=0.5$	$0.7500\approx\frac{3}{4}$	0.9417
$d=3$	Parallel	Sequential	Indefinite causal order
$k=1$	0	0	0
$k=2$	$\frac{1}{9}\approx 0.1111$	$0.1111\approx\frac{1}{9}$	$0.1111\approx\frac{1}{9}$

Figure: Optimal success probability of a heralded protocol that implements the inverse U_d^{-1} with k uses of U_d.

Final remarks

- Universal unitary inversion is possible!
  Parallel: $p \sim 1 - \frac{1}{k}$, Sequential: $p \sim 1 - \frac{1}{e^k}$

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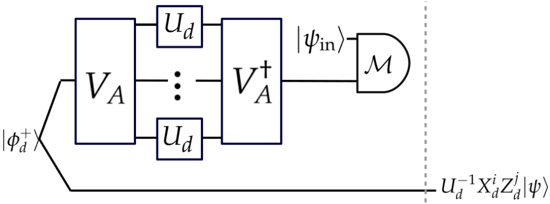
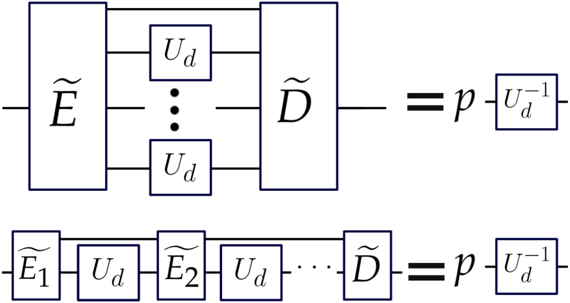
- Delayed input-state protocols:



Thank you!



$\widetilde{E}$
U_d
U_d
$\widetilde{D}$
$= p$
U_d^{-1}
$\widetilde{E_1}$
U_d
$\widetilde{E_2}$
U_d
$\cdots$
$\widetilde{D}$
$= p$
U_d^{-1}