Semi-device-independent certification of indefinite causal order

Marco Túlio Quintino

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Personal motivation

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- Find minimal assumptions for some particular claim

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- Claim more with some fixed set of assumptions

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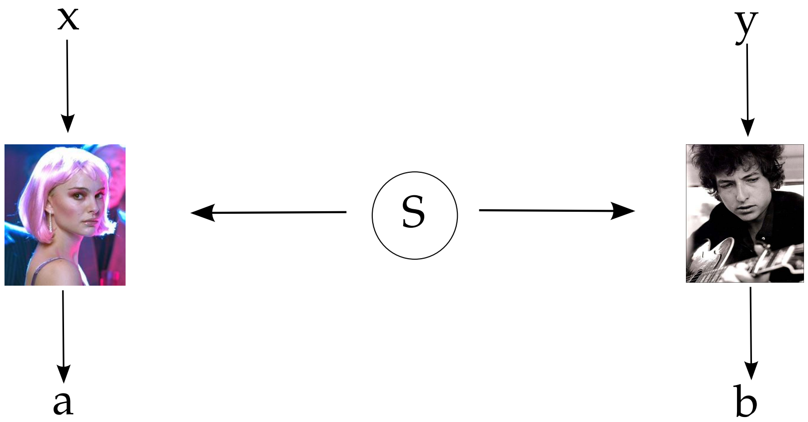
- Find minimal assumptions for some particular claim
- Claim more with some fixed set of assumptions
- Extracting maximal conclusions of physical experiments

Quantum entanglement

$$\rho_{AB} \neq \sum_{\lambda} \pi(\lambda) \rho_{A|\lambda} \otimes \rho_{B|\lambda}$$

Certification of entanglement

$$p(ab|xy) = \mathrm{tr}(\rho_{AB}\bar{A}_{a|x} \otimes \bar{B}_{b|y})$$



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- "Just" set $\bar{A}_{a|x}$ and $\bar{B}_{b|y}$ as trusted tomography complete measurements

Certification of entanglement

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- "Just" set $\bar{A}_{a|x}$ and $\bar{B}_{b|y}$ as trusted tomography complete measurements
- Or just violate some entanglement witness

Device-independent certification of entanglement

- If

$$p(ab|xy) \neq \sum_\lambda \pi(\lambda) p_A(a|x,\lambda) p_B(b|y,\lambda)$$

Device-independent certification of entanglement

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- Device-independent entanglement certification
- Bell inequalities are *device-independent entanglement witnesses*

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- Allows device-independent protocols
- Some entangled states cannot be certified in a device-independent way (*e.g.*, some Werner states)
- It is experimentally challenging

Semi-device-independent certification of entanglement

- Analyse $p(ab|xy)$ assuming Bob's measurements are known

H. M. Wiseman, S. J. Jones, A. C. Doherty, PRL (2007)

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$$p(ab|xy)^{\mathrm{sep}} = \mathrm{tr}(\rho_{AB}^{\mathrm{sep}} A_{a|x} \otimes \bar{B}_{b|y})$$

- Unsteerable assemblage:

$$p(ab|xy) = \mathrm{tr}(\sigma_{a|x}^{\mathrm{uns}} \bar{B}_{b|y})$$

$$\sigma_{a|x}^{\mathrm{uns}} := \mathrm{tr}_A(\rho_{AB}^{\mathrm{sep}} A_{a|x} \otimes I))$$

H. M. Wiseman, S. J. Jones, A. C. Doherty, PRL (2007)

Applications of semi-device-independent certification

- Steering interpretation

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  C. Branciard, E.G. Cavalcanti, S. Walborn, V. Scarani, H. M. Wiseman, PRA (2012)

Applications of semi-device-independent certification

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- One-side device independent protocols
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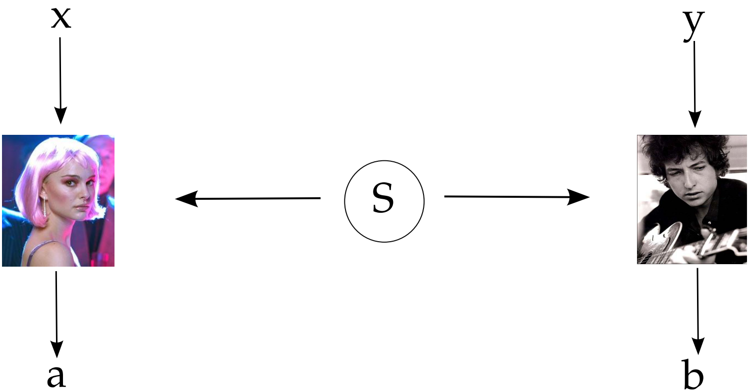
- Provides technical tools and good insights for the device-independent case

Abstract view on bipartite quantum systems



$$p(ab|xy) = f(A_{a|x}, B_{b|y})$$

Abstract view on bipartite quantum states

The most general bi-linear function $f(A_{a|x}, B_{b|y})$ that extract valid probability distributions from quantum measurements[1] given by POVMs $A_{a|x} \in L(A)$ and $B_{b|y} \in L(B)$ is given by

$$\mathrm{tr}(\rho_{AB} A_{a|x} \otimes B_{b|y}),$$

where $\rho_{AB} \in L(A \otimes B)$ is a quantum state.

[1] And their trivial extensions.

More general scenarios

- And what if the parties can have a causal relation and exchange quantum systems?

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- Instead of POVMs, Alice and Bob have quantum instruments

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- And what if the parties can have a causal relation and exchange quantum systems?
- Instead of POVMs, Alice and Bob have quantum instruments
- Quantum instruments have a "classical output" and a "quantum output"

Process matrix

The most general bi-linear function $f(A_{a|x}, B_{b|y}) = \mathrm{tr}(A_{a|x} \otimes B_{b|y} W)$ that extract valid probability distributions from quantum instruments[2] given by the Choi operators $A_{a|x} \in L(A_I \otimes A_O)$ and $B_{b|y} \in L(B_I \otimes B_O)$

[2] And their trivial extensions.

Process matrix

- If $\mathrm{tr}(A_{a|x} \otimes B_{b|y} W) = p(ab|xy)^{\mathrm{NS}}$ is non-signalling, the process is a quantum state $W = \rho_{A_I,B_I} \otimes I_{A_O,B_O}$

Process matrix

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- If $\mathrm{tr}(A_{a|x} \otimes B_{b|y} W) = p(ab|xy)^{A<B}$, the process $W^{A<B}$ is a channel with memory/comb/ordered circuit

Quantum Channels with Memory (2005)
D. Kretschmann, R. F. Werner

Transforming quantum operations: quantum supermaps (2008)

G. Chiribella, G. M. D'Ariano, P. Perinotti enditemize

Causally separable process matrix

Causally separable:

$$W^{\text{sep}} = qW^{A<B} + (1-q)W^{B<A},$$

for some probability $0 \leq q \leq 1$.

Quantum correlations with no causal order (2011)
O. Oreshkov, F. Costa, C. Brukner

Process matrix

The most general bi-linear function $f(A_{a|x}, B_{b|y})$ that extract valid probability distributions from quantum instruments[3] given by the Choi operators $A_{a|x} \in L(A_I \otimes A_O)$ and $B_{b|y} \in L(B_I \otimes B_O)$ is

$$p(ab|xy) = \mathrm{tr}(W\ A_{a|x} \otimes B_{b|y}),$$

where $W \in L(A_I \otimes A_O \otimes B_I \otimes B_O)$ is a process matrix.

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[3] And their trivial extensions.

Process matrix

$W \in L(A_I \otimes A_O \otimes B_I \otimes B_O)$ is a process matrix iff

$$
\begin{aligned}
W &\geq 0; \\
{}_{A_I A_O}W &= {}_{A_I A_O B_O}W; \\
{}_{B_I B_O}W &= {}_{B_I B_O A_O}W; \\
W + {}_{A_O B_O}W &= {}_{B_O}W + {}_{A_O}W; \\
\mathrm{tr}(W) &= d_{A_O} d_{B_O}.
\end{aligned} \tag{1}
$$

Device-dependent certification non-causal separability

Given a set of probabilities $p(ab|xy)$ and instruments $\bar{A}_{a|x}$,$\bar{B}_{b|y}$

$$p(ab|xy) = \mathrm{tr}(W\bar{A}_{a|x} \otimes \bar{B}_{b|y})$$

Causal witness:

Witnessing causal nonseparability (2015)

M. Araújo, C. Branciard, F. Costa, A. Feix, C. Giarmatzi, Č. Brukner

Device-independent certification non-causal separability

Given a set of probabilities $p(ab|xy)$

$$p(ab|xy) = \text{tr}(W\ A_{a|x} \otimes B_{b|y})$$

Causal games:
The simplest causal inequalities and their violation
C. Branciard, M. Araújo, A. Feix, F. Costa, Č. Brukner

Semi-device-dependent certification non-causal separability

How about semi-device independent?

Semi-device-dependent certification non-causal separability

Given a set of probabilities $p(ab|xy)$ and instruments $\bar{B}_{b|y}$ does there exist a causally separable process matrix W^{sep} and instruments $A_{a|x}$

$$p(ab|xy) = \text{tr}(WA_{a|x} \otimes \bar{B}_{b|y})$$

Semi-device-dependent certification non-causal separability

Device-dependent

Given quantities	Variables
$\{p^Q(ab\|xy)\}$ $\{\overline{A}_{a\|x}\}, \{\overline{B}_{b\|y}\}$	W

Device-independent

Given quantities	Variables
$\{p^Q(ab\|xy)\}$	$d_{A_I}, d_{A_O}, d_{B_I}, d_{B_O}$ $\{A_{a\|x}\}, \{B_{b\|y}\}$ W

Semi-device-independent

Given quantities	Variables
$\{p^Q(ab\|xy)\}$ $\{\overline{B}_{b\|y}\}$	d_{A_I}, d_{A_O} $\{A_{a\|x}\}$ W

Useful tools

- Process assemblage:

$$w_{a|x} := \mathrm{tr}(W A_{a|x} \otimes I)$$

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$$w_{a|x} := \mathrm{tr}\left(W A_{a|x} \otimes I \right)$$

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- Abstract causally separable assemblage:
  $w^{A<B}_{a|x}$ such that $\mathrm{tr}(w^{A<B}_{a|x} B_{b|y}) = p^{A<B}(ab|xy)$

Useful tools

- Process assemblage:

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- Abstract causally separable assemblage:

$$w^{\text{asep}}_{a|x} := q w^{A<B}_{a|x} + (1-q) w^{B<A}_{a|x}$$

Main result 1

Theorem

All abstract causally separable process assemblage can be realised by causally separable process matrix. That is, $\exists W^{sep}, \{A_{a|x}\}$ *such that*

$$w_{a|x}^{asep} = \mathrm{tr}(W^{sep} A_{a|x} \otimes I) = w_{a|x}^{sep}.$$

Moreover, deciding if a process assemblage is causally separable can be phrased as a semi-definite program.

“No analogue to Schrödinger’s theorem”

Theorem

There exists an abstract assemblage with no process matrix realisation.

$$w_{a|x} \neq \mathrm{tr}(W^A_{a|x} \otimes I)$$

General statements about the process matrix?

How about statements for the process matrix?

Main result 2

Theorem

Let W be a bipartite process matrix. If W^{T_B} is causally separable, W is semi-device independent causally separable

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Theorem (This work)

Let W be a bipartite process matrix. If W^{T_B} is causally separable, W is semi-device independent causally separable.

Theorem (Feix *et al*)

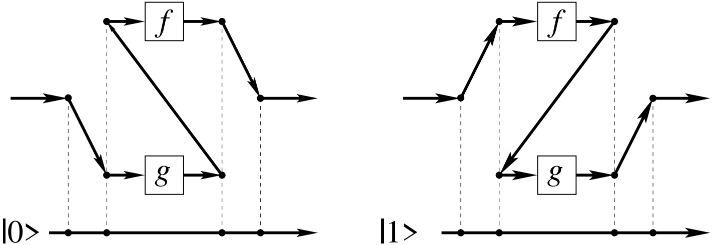
Let W be a bipartite process matrix. If W^{T_B} is causally separable, W is device independent causally separable.

Causally nonseparable processes admitting a causal model (2016)
A. Feix, M. Araújo, Č. Brukner

Too abstract. . .

OK, nice. But. . . how do we "realise" such indefinite causal order processes ??

Quantum Switch



Quantum computations without definite causal structure (2013)
G. Chiribella, G. M. D'Ariano, P.Perinotti, B. Valiron

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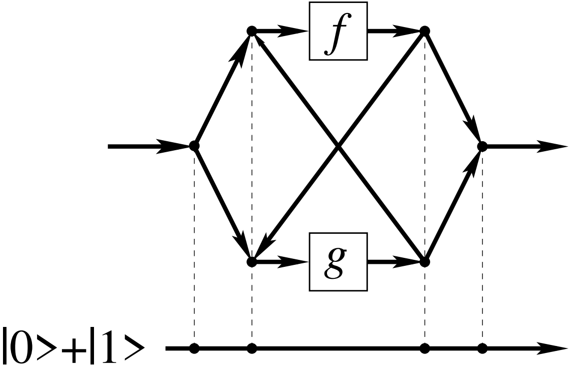

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Tripartite process matrices

- The quantum switch a process with indefinite causal order.

Tripartite process matrices

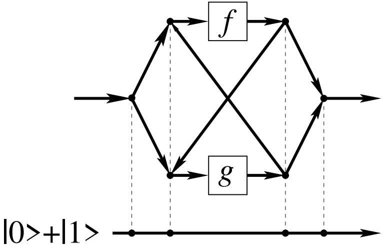
- The quantum switch a process with indefinite causal order.
- But cannot lead to device-independent certification. . .

Witnessing causal nonseparability (2015)

M. Araújo, C. Branciard, F. Costa, A. Feix, C. Giarmatzi, Č. Brukner

Tripartite process matrices

Alice	Bob	Charlie
T	T	T
T	T	U
T	U	T
T	U	U
U	T	T
U	T	U
U	U	T
U	U	U



Main result 3

Theorem

In the UUT scenario, the quantum switch is semi-device-independent causally separable.

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Theorem (This work)

In the UUT scenario, the quantum switch is semi-device-independent causally separable.

Theorem (Araújo et al)

The quantum switch is device-independent causally separable.

Witnessing causal nonseparability (2015)
M. Araújo, C. Branciard, F. Costa, A. Feix, C. Giarmatzi, Č. Brukner

Tripartite process matrices

- How about other scenarios?

Tripartite process matrices

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- Noisy Switch:

$$W_{\text{switch}}(r) = (1-r)W_{\text{switch}} + r\,\frac{I}{4}$$

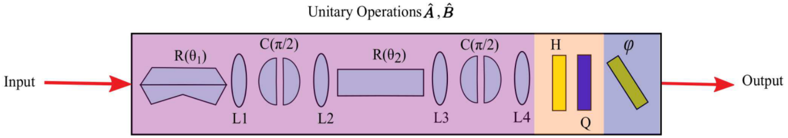
Critical visibility r indefinite causal order

<table>
<tr><td colspan="2">TTT
$r^* = 0.6118$
Noncausal</td></tr>
<tr><td>UTT
$r^* \geq 0.1802$
Noncausal</td><td>TTU
$r^* \geq 0.5687$
Noncausal</td></tr>
<tr><td>UUT
$r^* = 0$
Causal</td><td>TUU
$r^* \geq 0.1621$
Noncausal</td></tr>
<tr><td colspan="2">UUU
$r^* = 0$
Causal</td></tr>
</table>

Applications

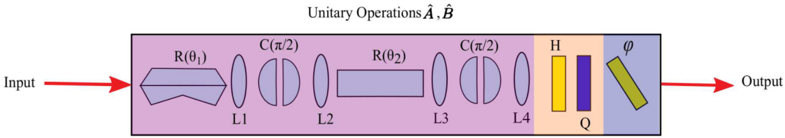
Analysing real experiments

Analysing previous experiments



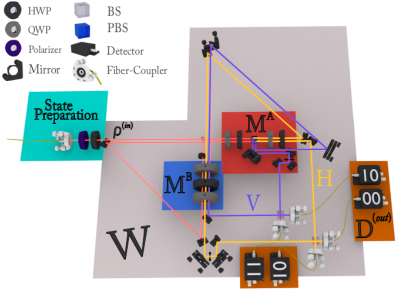
Indefinite Causal Order in a Quantum Switch, PRL (2018)
K. Goswami, C. Giarmatzi, M. Kewming, F. Costa, C. Branciard, J. Romero, A. G. White

Analysing previous experiments



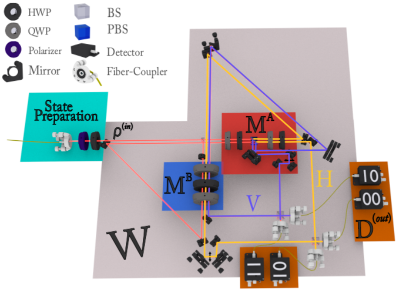
With these unitaries, indefinite causal order can be certified without assuming Charlie performs σ_X
(Robustness: $r = 0.1989$)

Analysing previous experiments



Experimental Verification of an Indefinite Causal Order, Sci. Adv. (2017)
G. Rubino, L. A. Rozema, A. Feix, M. Araújo, J. M. Zeuner, L. M. P., Č. Brukner, P. Walther

Analysing previous experiments



With these instruments and states, indefinite causal order can be certified without assuming Charlie performs σ_X
(Robustness: $r = 0.2300$)

Final remarks

- Simple framework to analyse indefinite causal order based on trusted instruments

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- Question: Can we have other interpretations? (maybe related to EPR-steering?)
- Question: Are these ideas/methods useful for some computational/communication task?

Thank you!