# Entangled States Admitting a Local Hidden Variable Model For Sequential Measurements

Marco Túlio Quintino

Université de Genève

31st March 2016

UNIVERSITÉ DE GENÈVE

FNSNF

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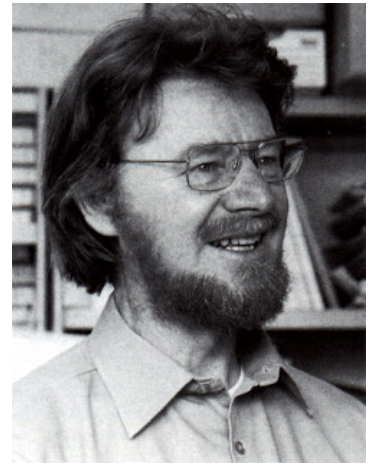
FONDO NAZIONALE SVIZZERO
SWISS NATIONAL SCIENCE FOUNDATION

Joint with: J. Bowles, F. Hirsch, N. Brunner (In preparation)

# Entanglement *Vs.* Nonlocality

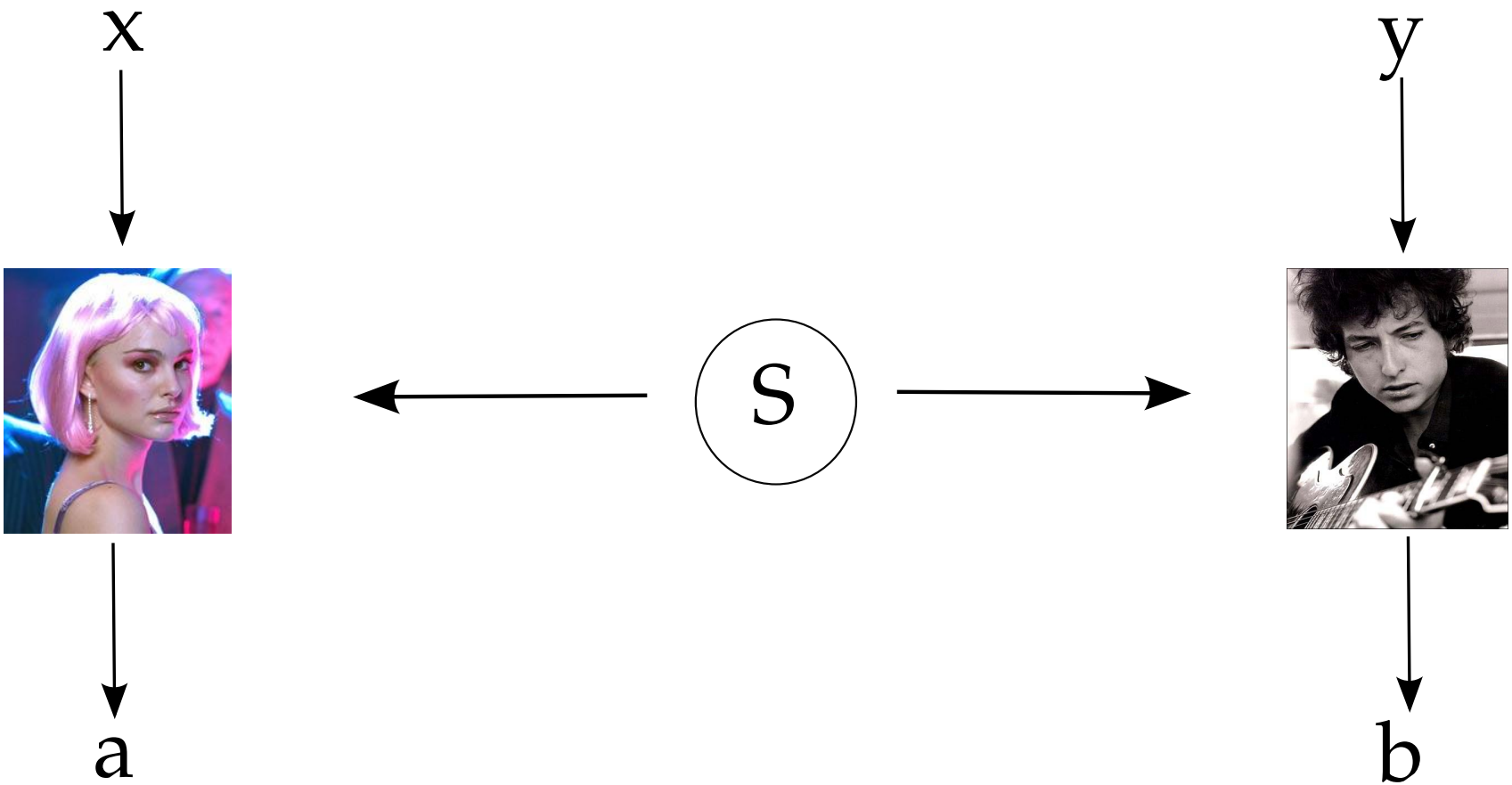
*Vs.*



# Entanglement

$$\rho_{AB} \neq \int \pi(\lambda) \rho_A^\lambda \otimes \rho_B^\lambda \, \mathrm{d}\lambda$$

# Bell Scenario



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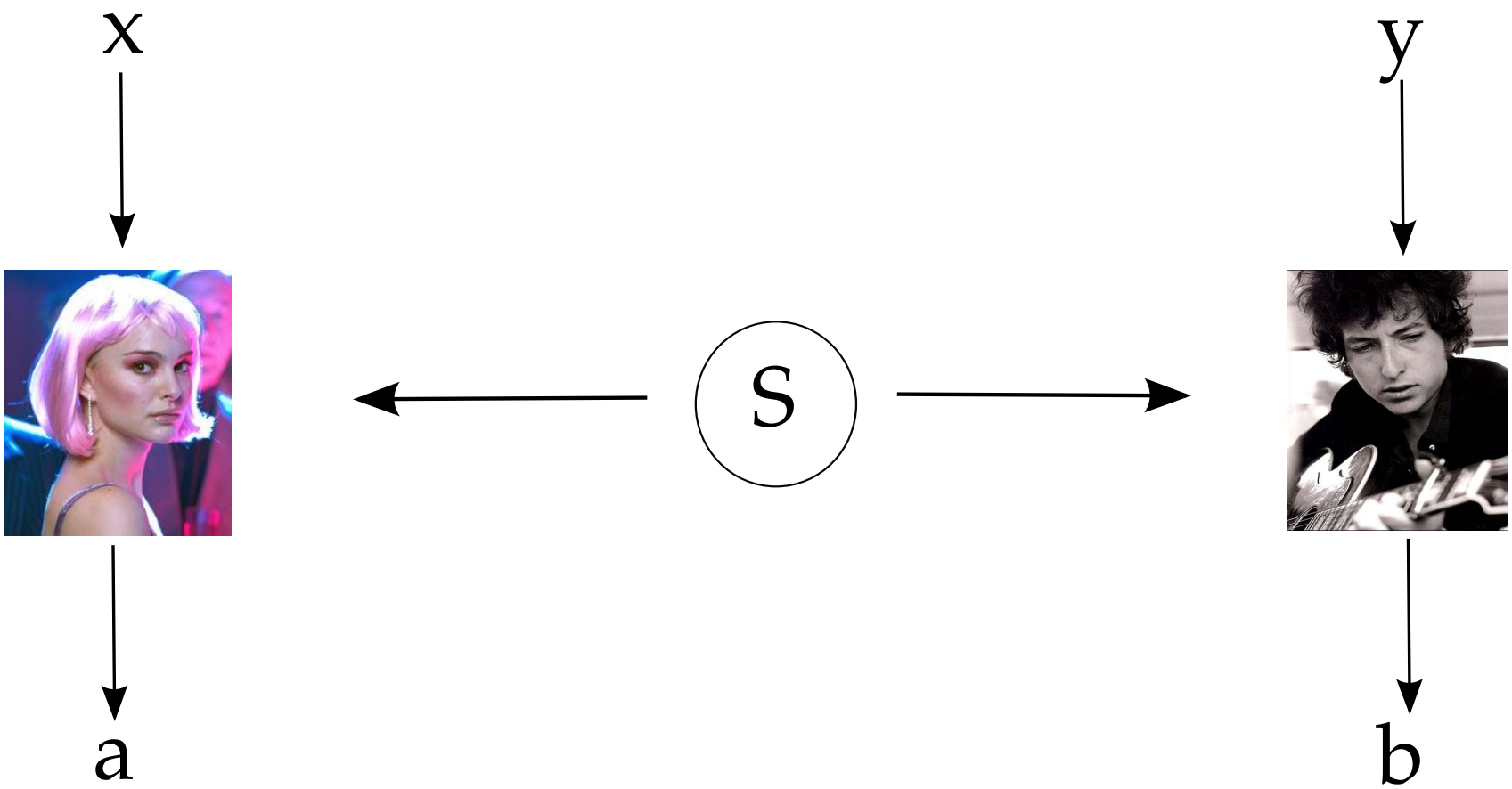
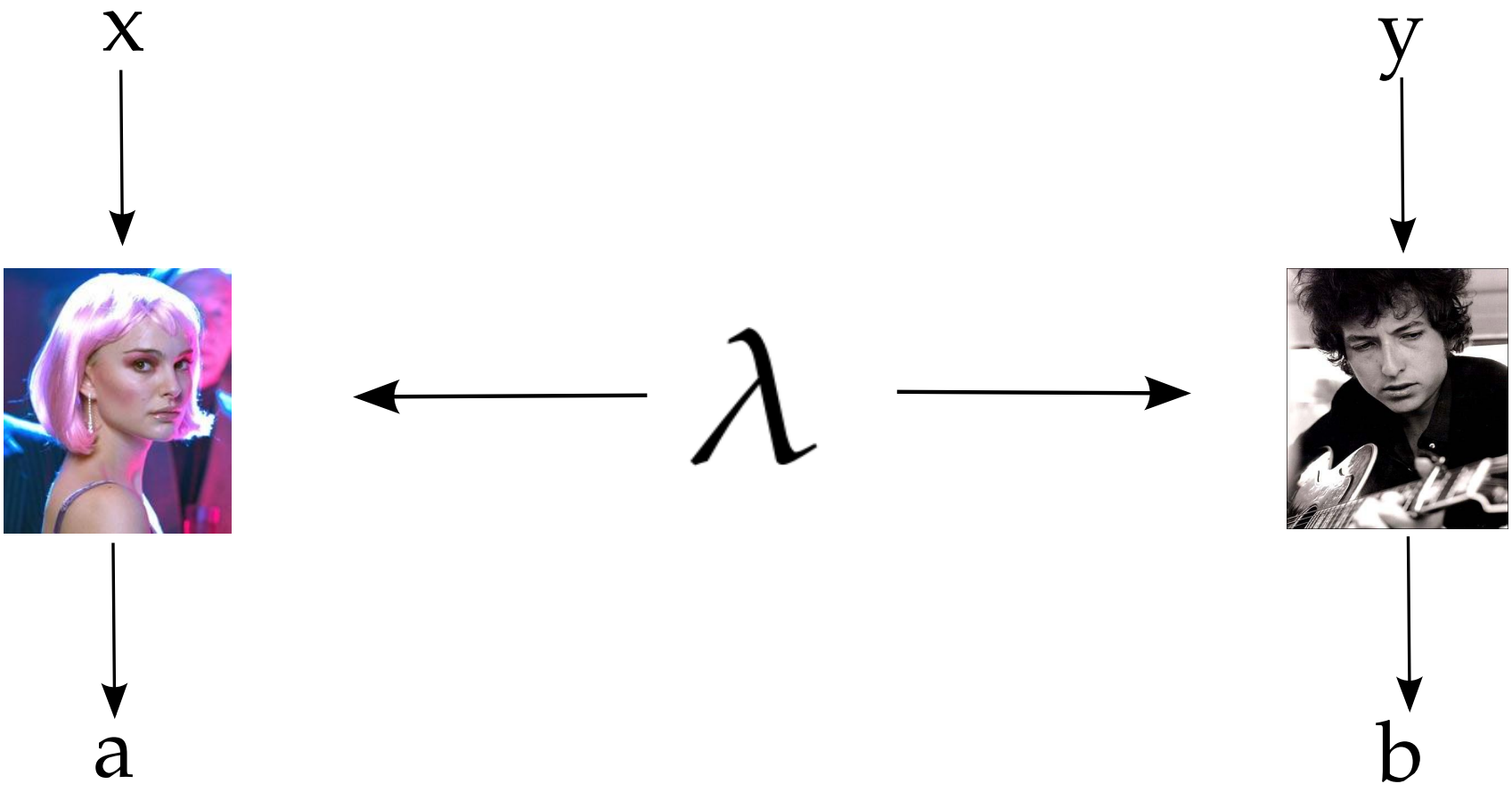



$$p(ab|xy)$$

# Locality



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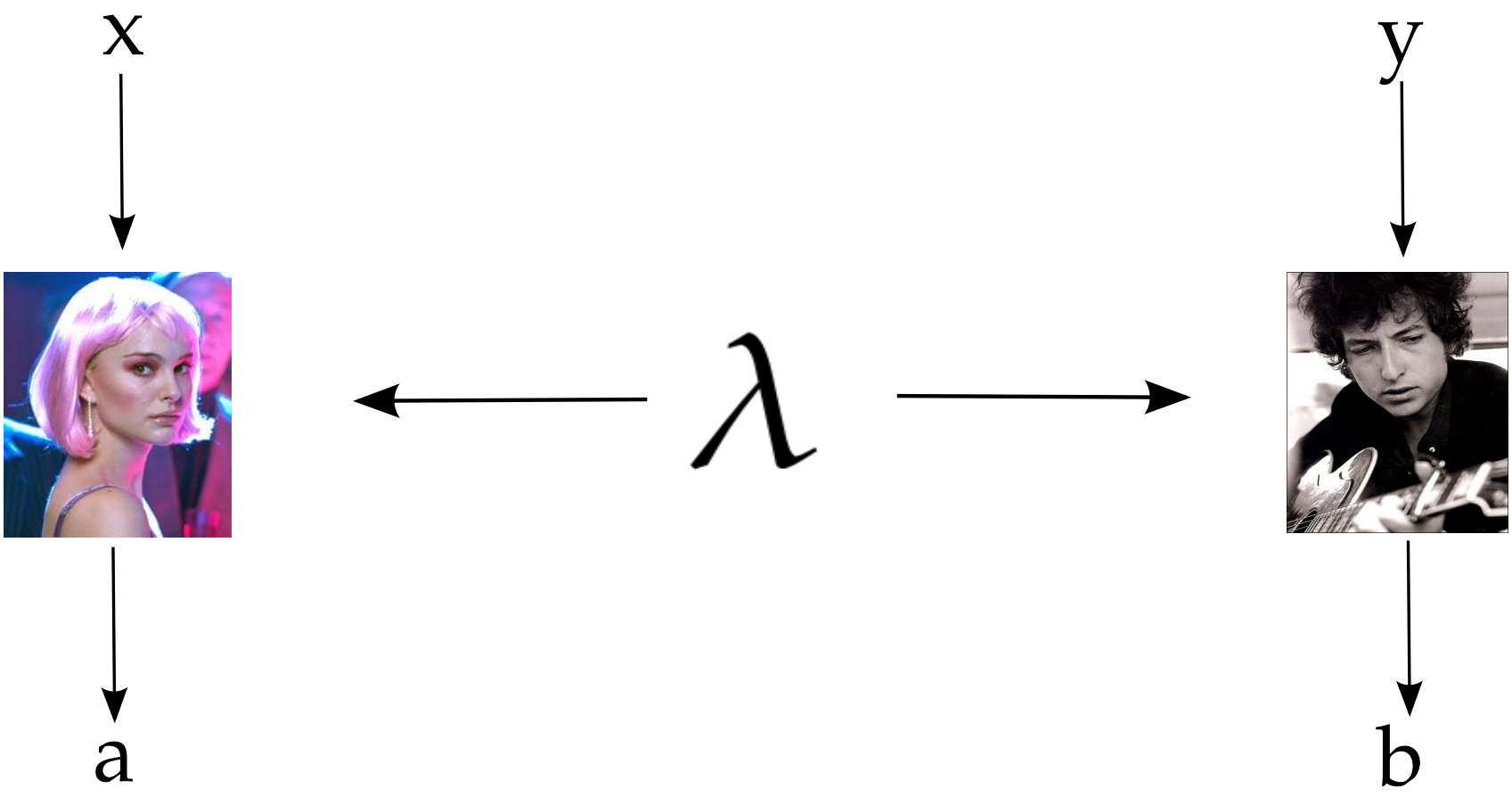
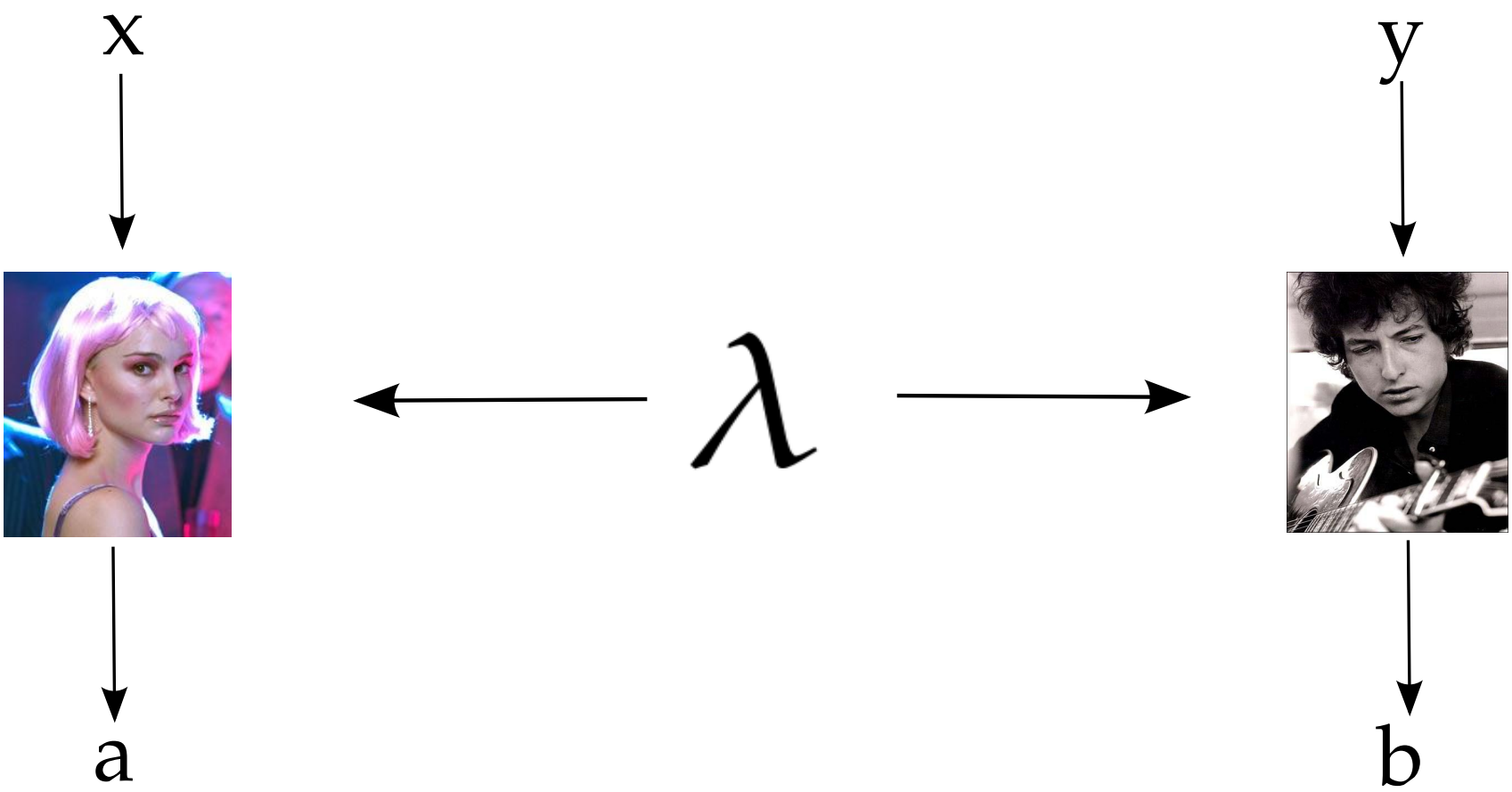



- Common cause:

$$p(ab|xy\lambda) = p(a|x\lambda)p(b|y\lambda)$$

# Locality



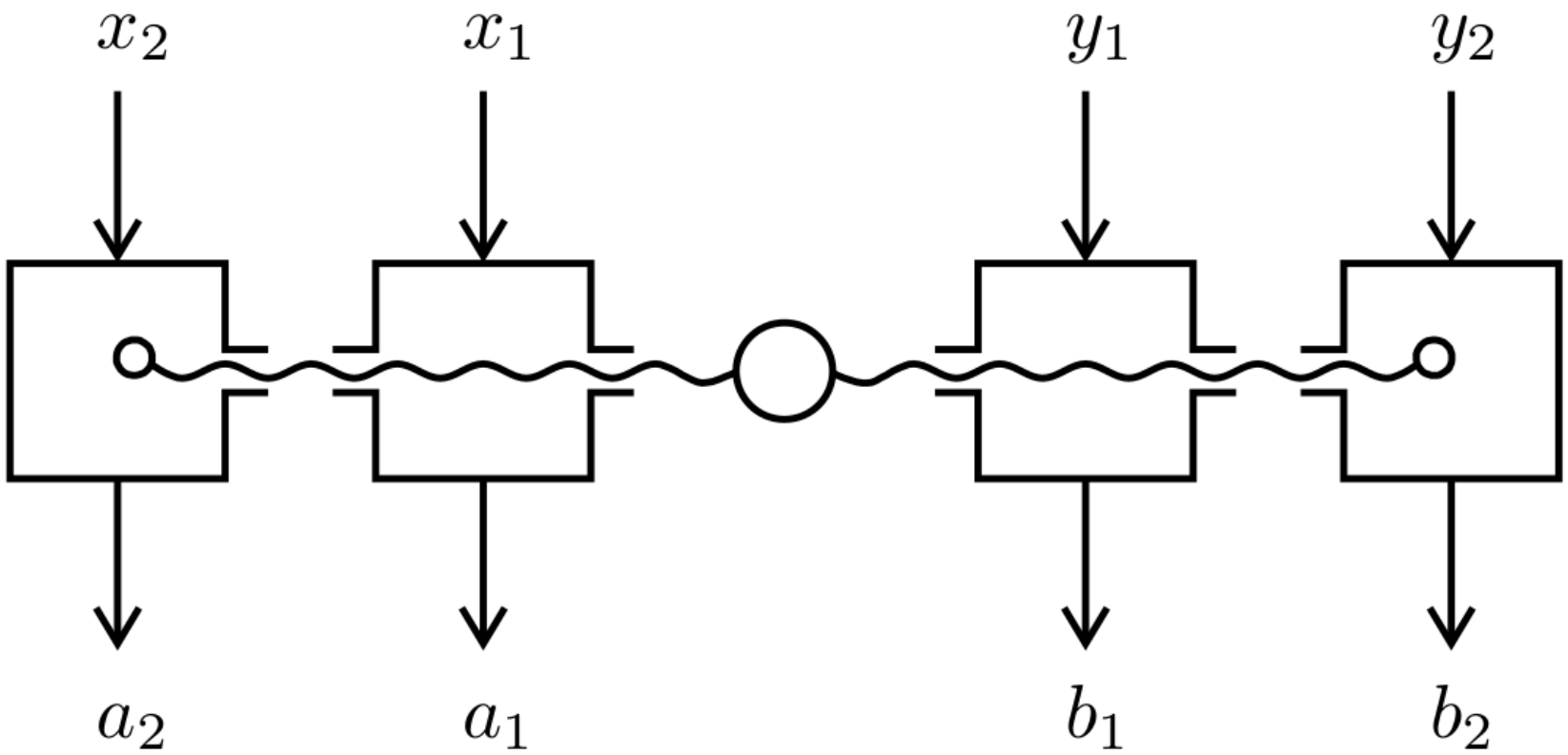

- Common cause:

$$p(ab|xy\lambda) = p(a|x\lambda)p(b|y\lambda)$$

- Locality:

$$p(ab|A_x B_y) = \int \pi(\lambda)p(a|x\lambda)p(b|y\lambda)\,\mathrm{d}\lambda$$

# Bell Scenario with sequential measurements




Picture: Nonlocality in sequential correlation scenarios
R. Gallego, L. Würflinger, R. Chaves, A. Acín and M. Navascués
New J. Phys. 16 (2014) 033037

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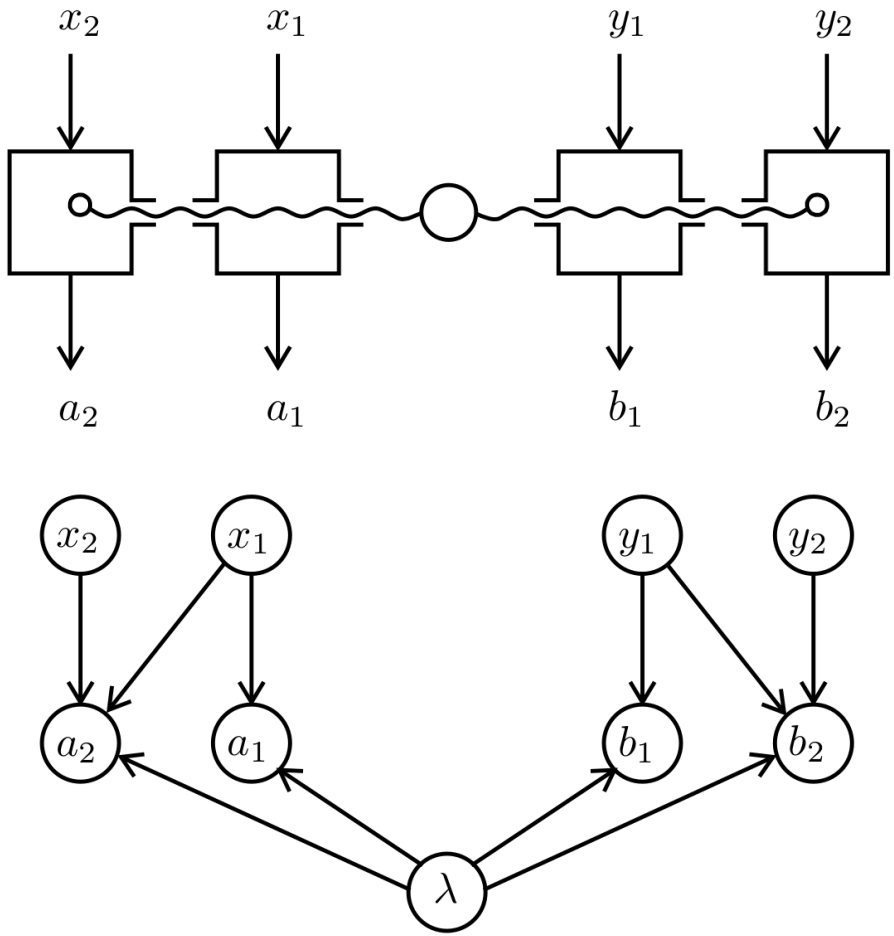


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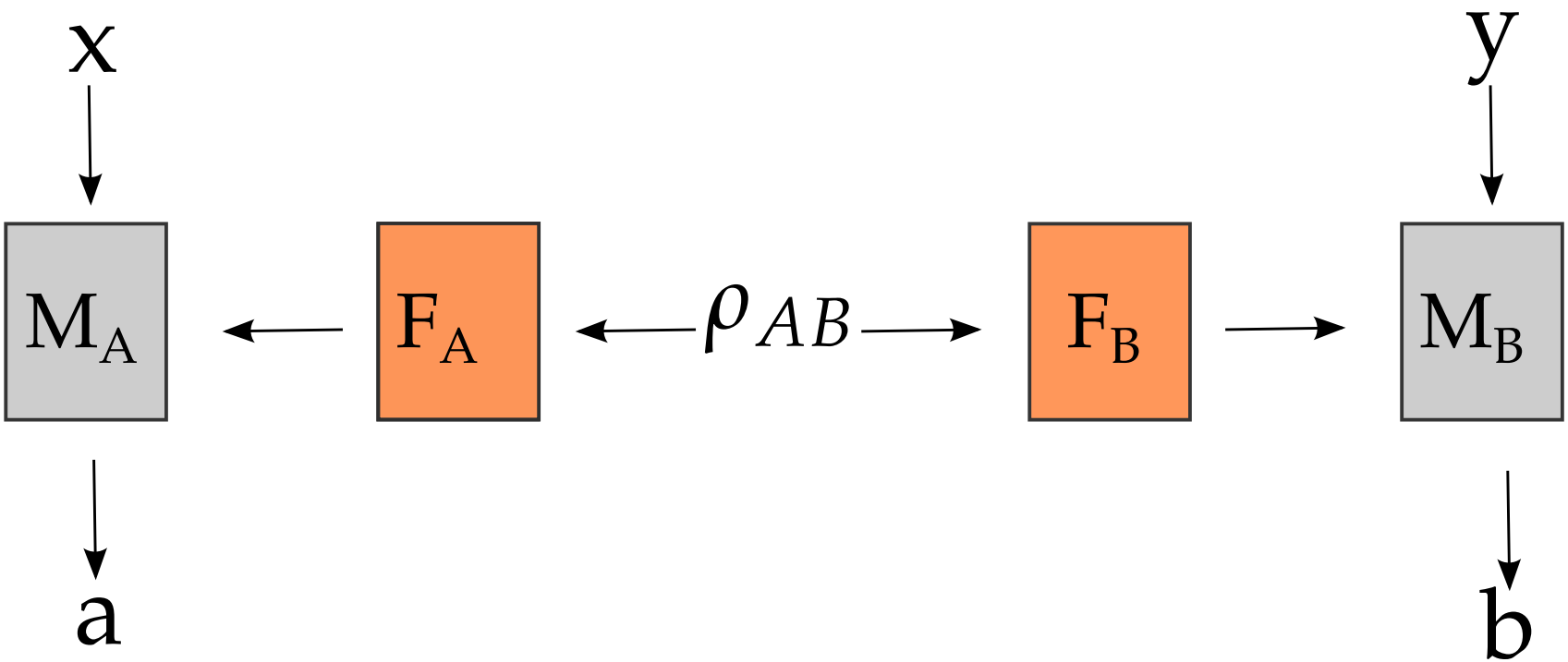
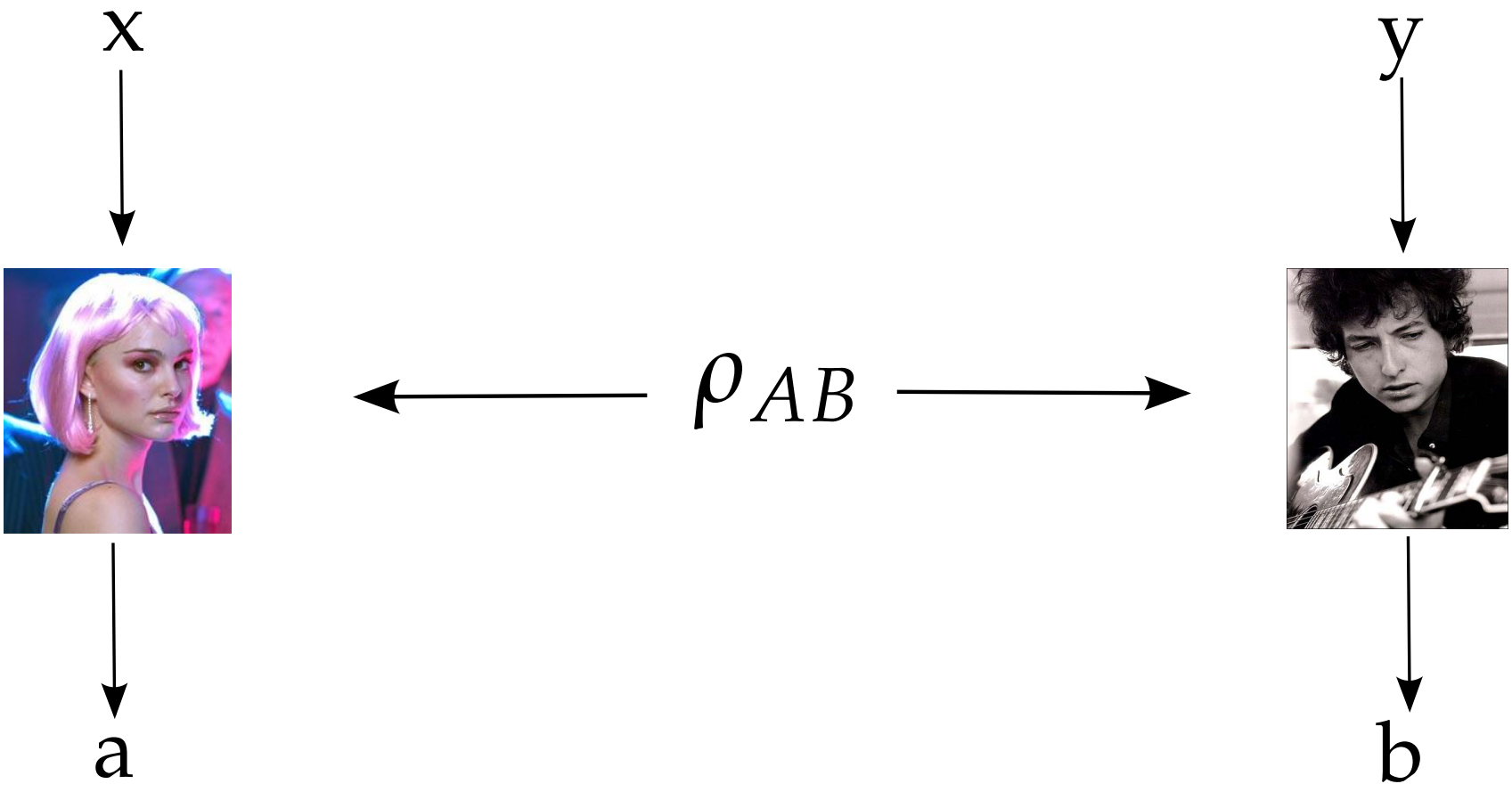


# Quantum Experiments



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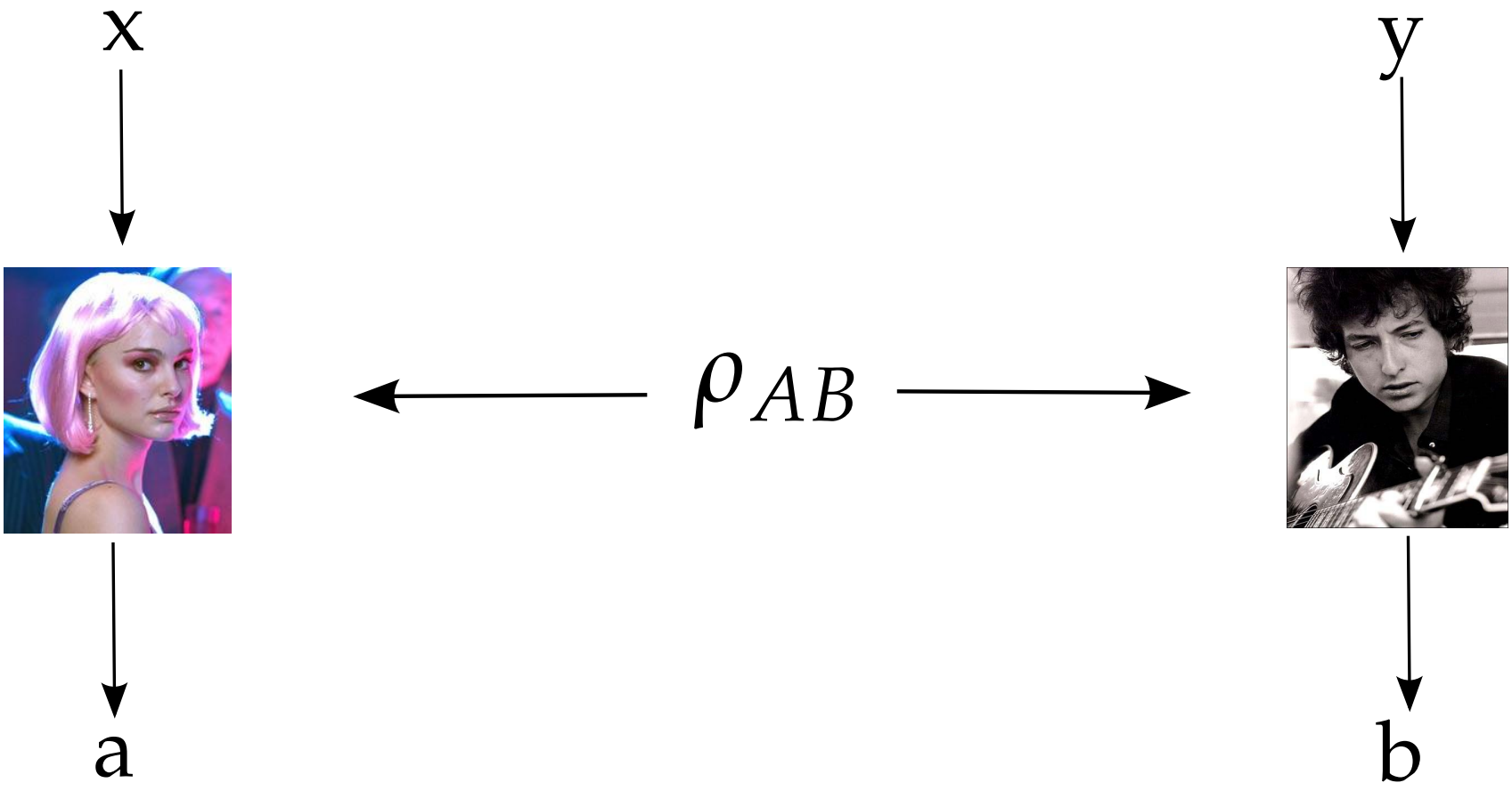


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$$p(ab|xy) = \mathrm{tr}\left(\rho_{AB} A_a^x \otimes B_b^y\right)$$

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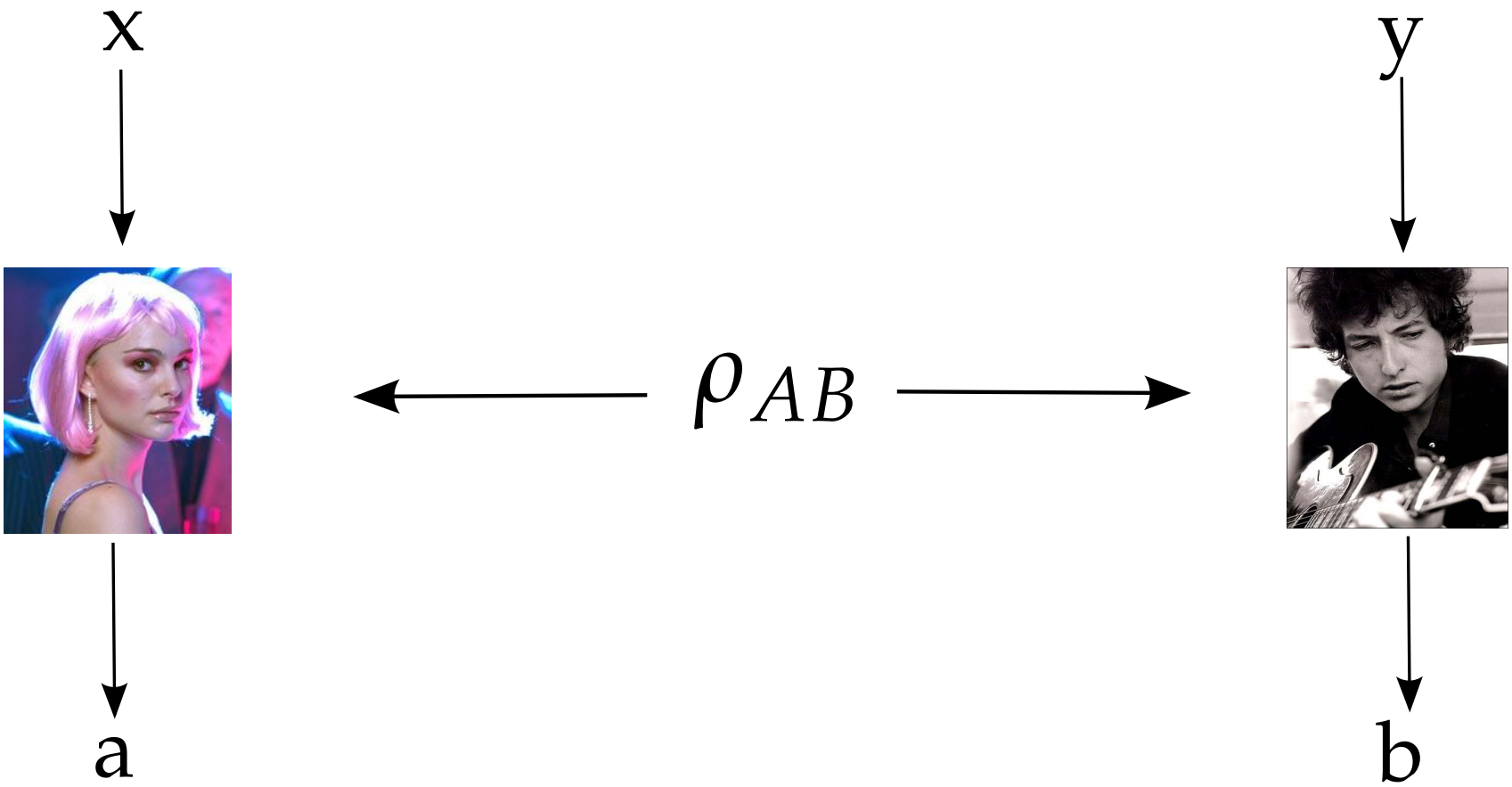


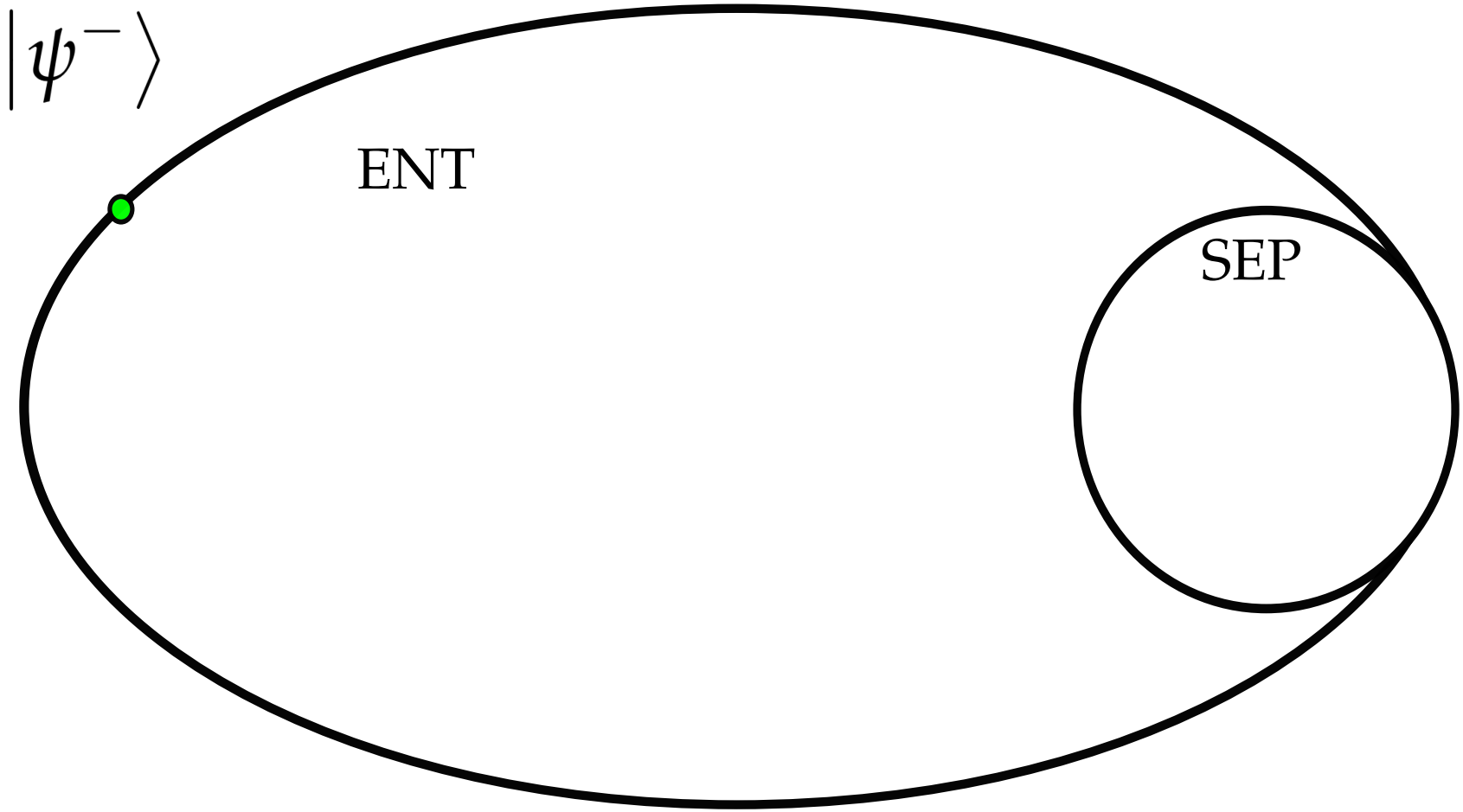

- Quantum probabilities:

$$p(ab|xy) = \mathrm{tr}\left(\rho_{AB} A_a^x \otimes B_b^y\right)$$

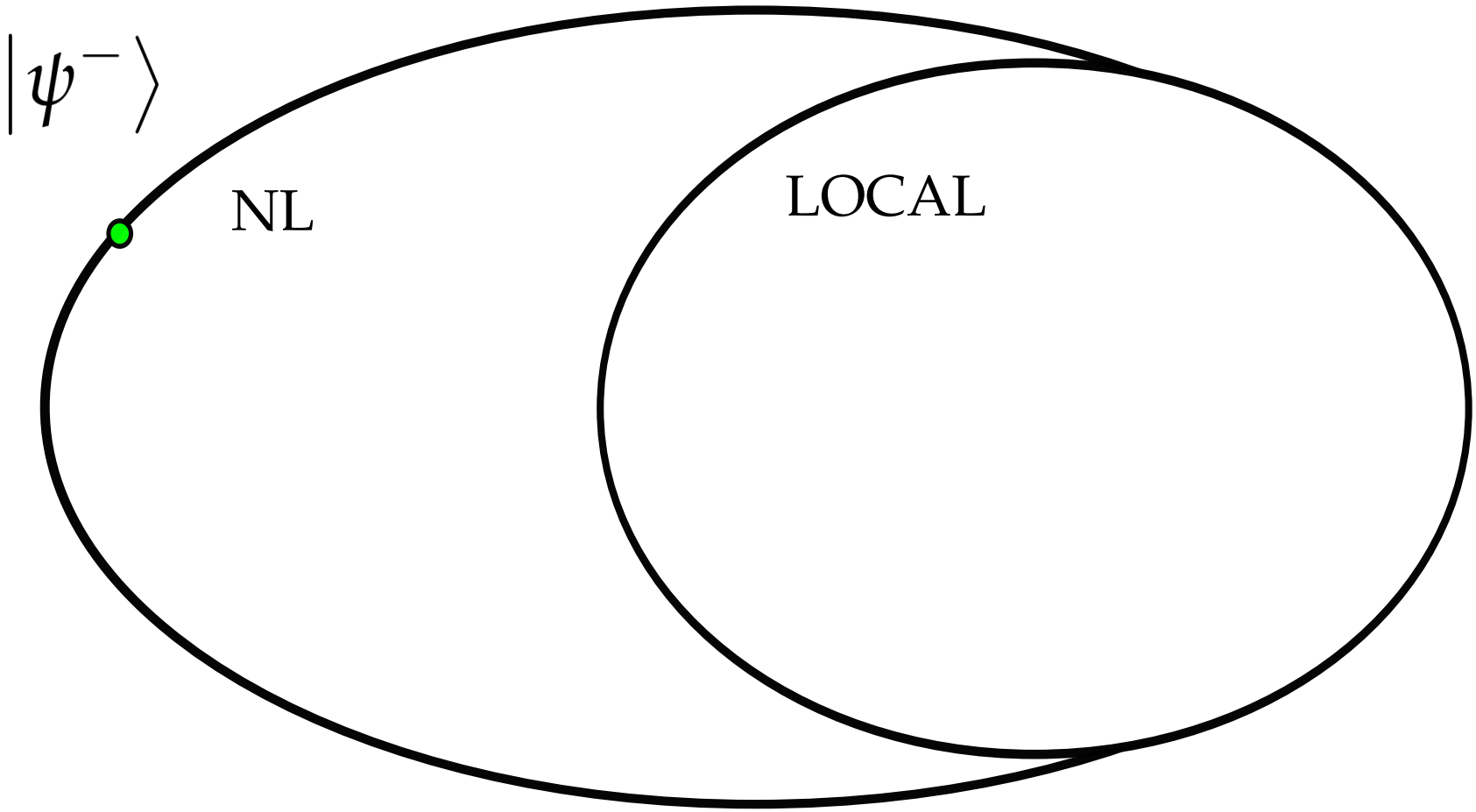
- "Quantum theory is nonlocal" (Bell 64)

$$\mathrm{tr}\left(\psi_{AB} A_a^x \otimes B_b^y\right) \neq \int \pi(\lambda) p(a|x\lambda) p(b|y\lambda)\, \mathrm{d}\lambda$$

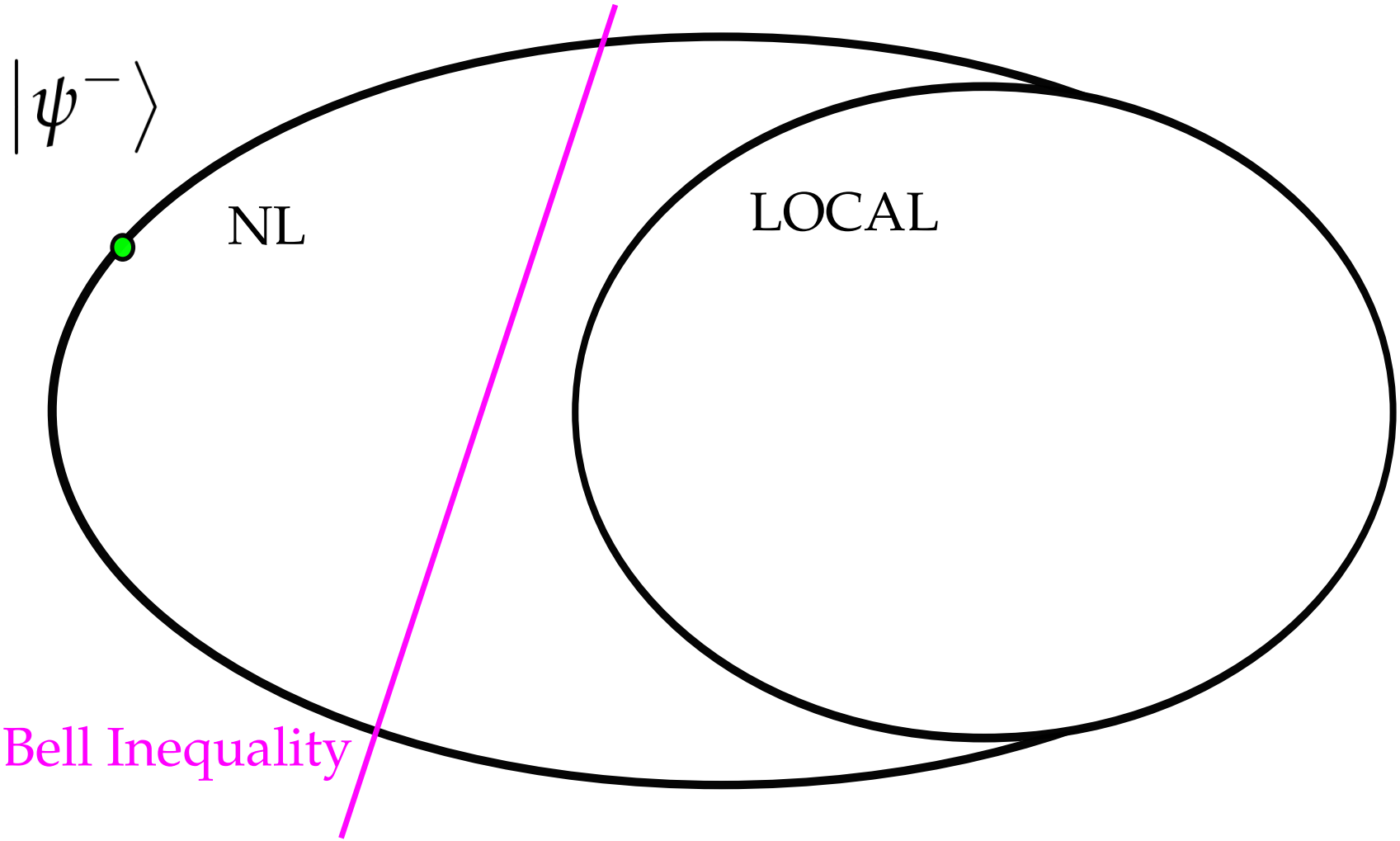
# Separable/Entanglement



# Local/Nonlocal



# Bell inequalities



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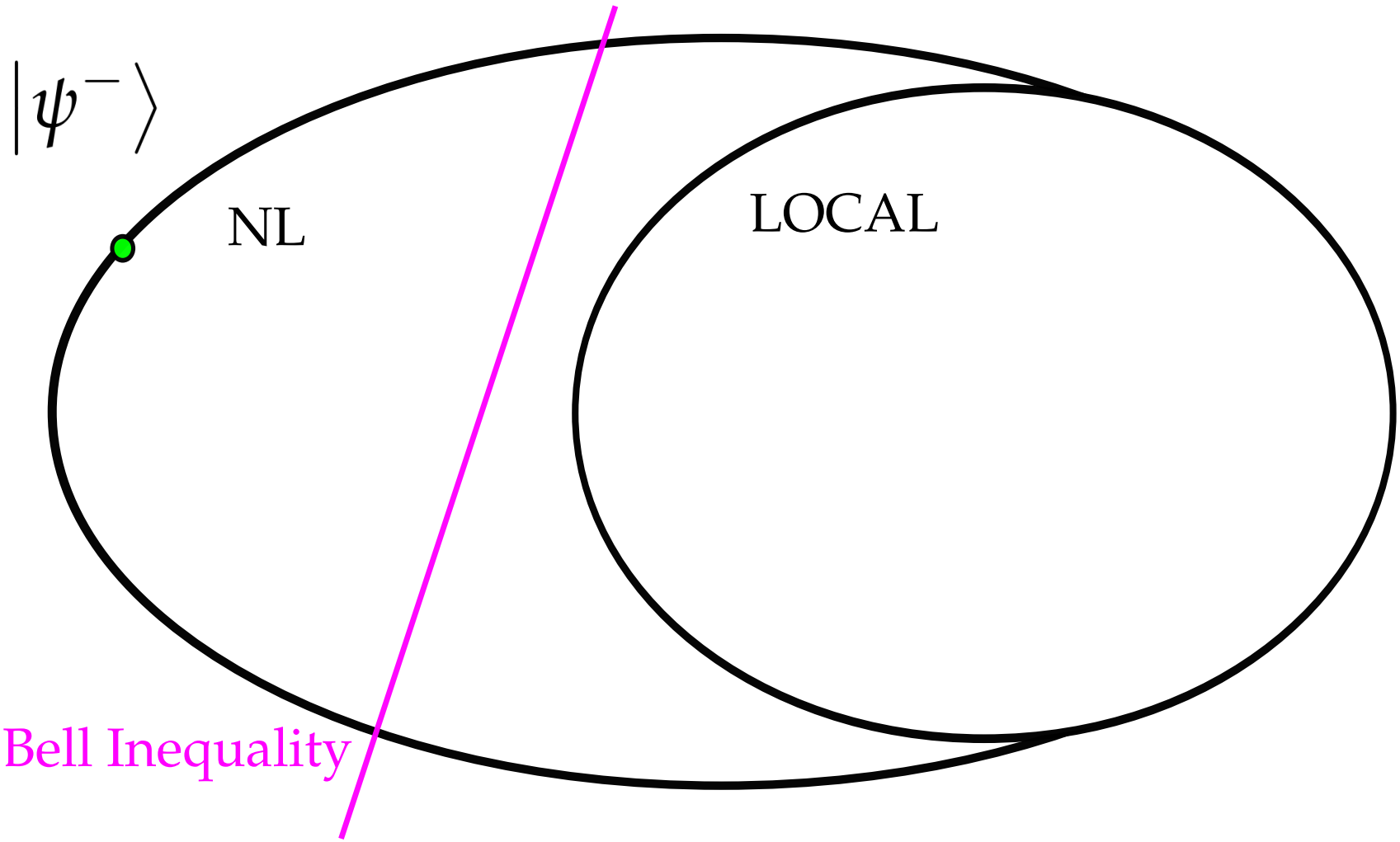
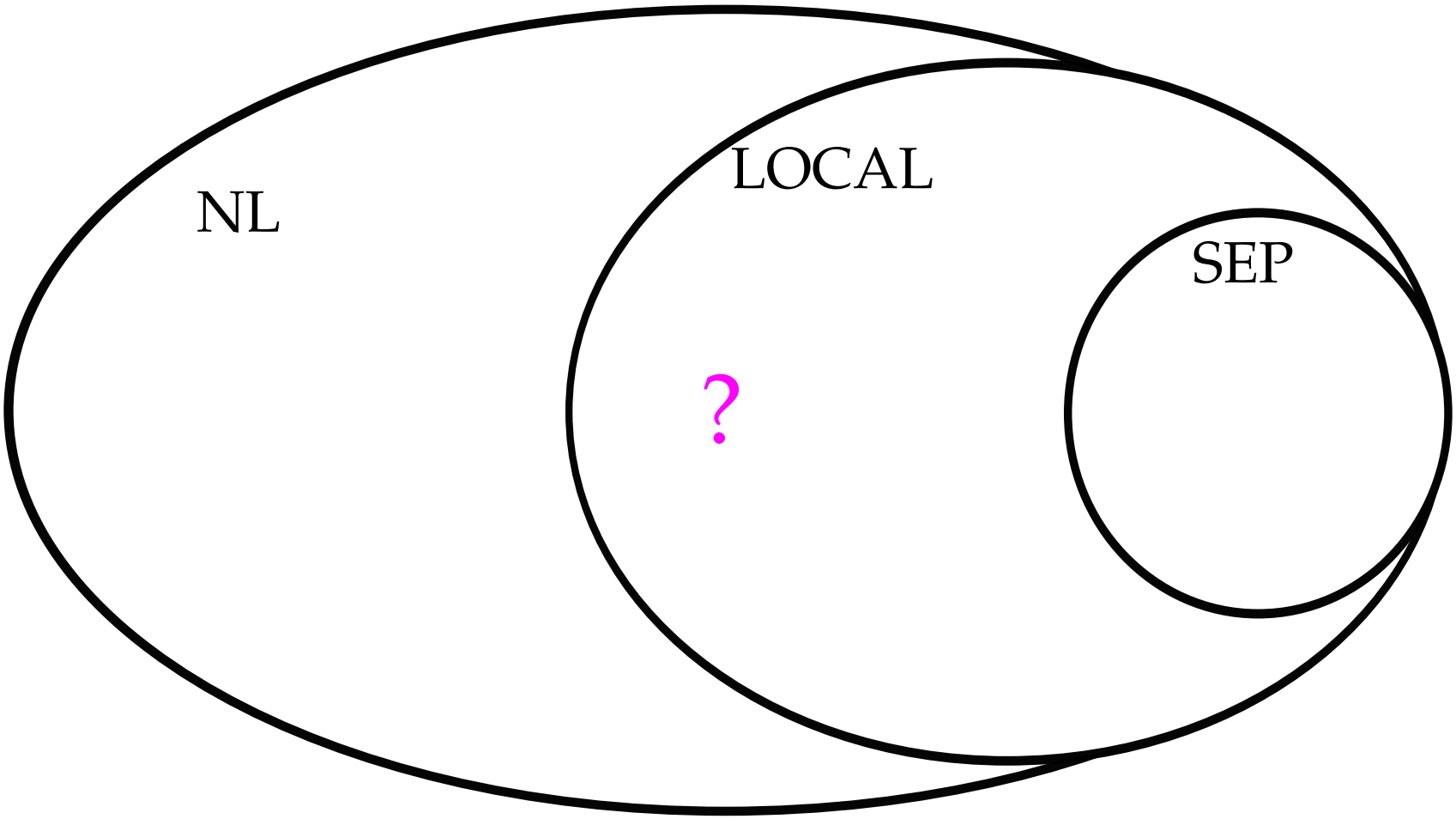



*e.g.:*

$$CHSH = A_0 \otimes B_0 + A_0 \otimes B_1 + A_1 \otimes B_0 - A_1 \otimes B_1$$

# Separable $\stackrel{?}{=}$ Local



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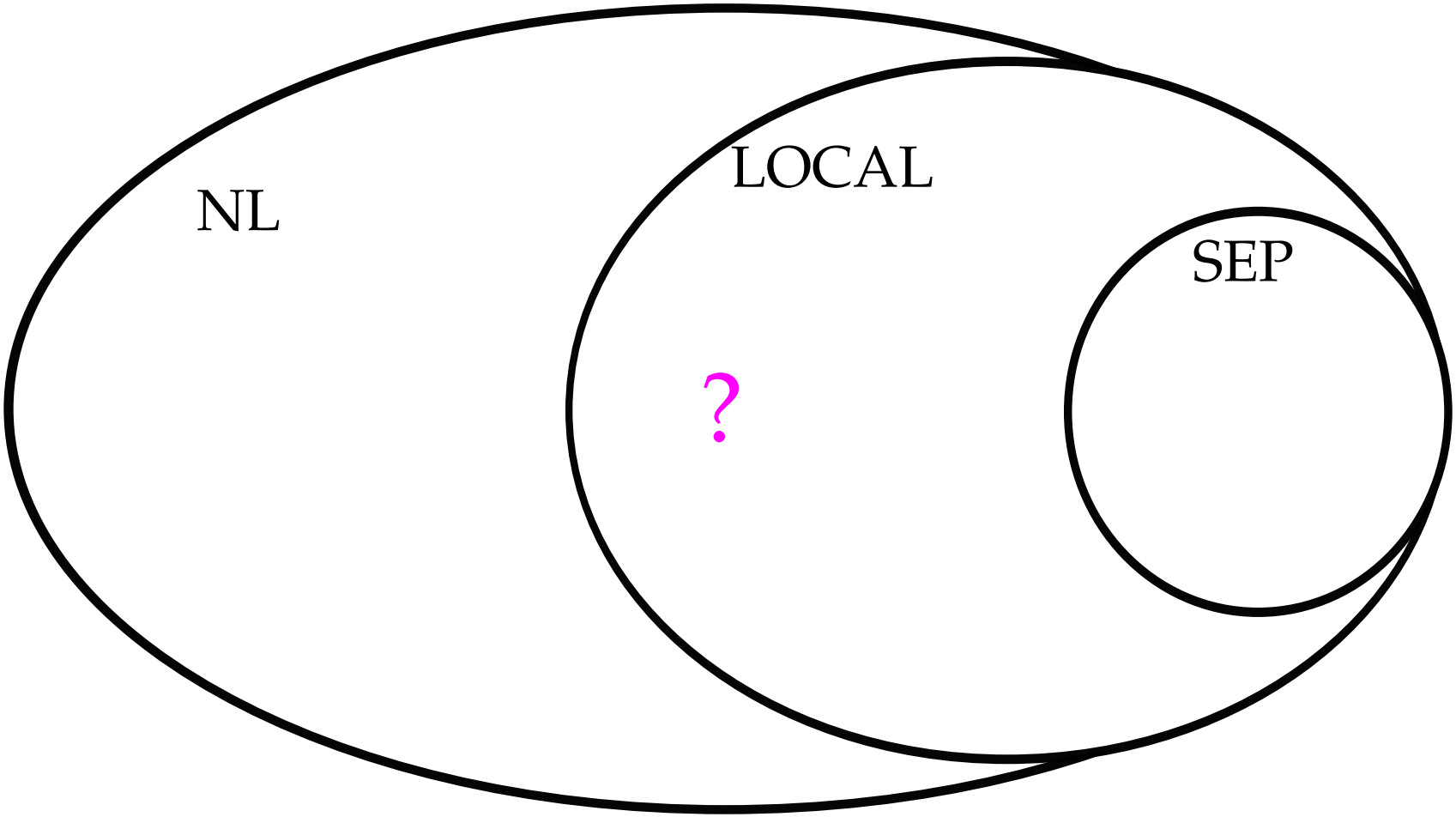


$$\rho_{AB} \neq \int \pi(\lambda)\rho_A^\lambda \otimes \rho_B^\lambda \, \mathrm{d}\lambda$$

$$p(ab|xy) \neq \int \pi(\lambda)p(a|x\lambda)p(b|y\lambda) \, \mathrm{d}\lambda$$

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- Pure states: Yes (Gisin 91, Popescu and Rohrlich 92)
- Mixed states: ???

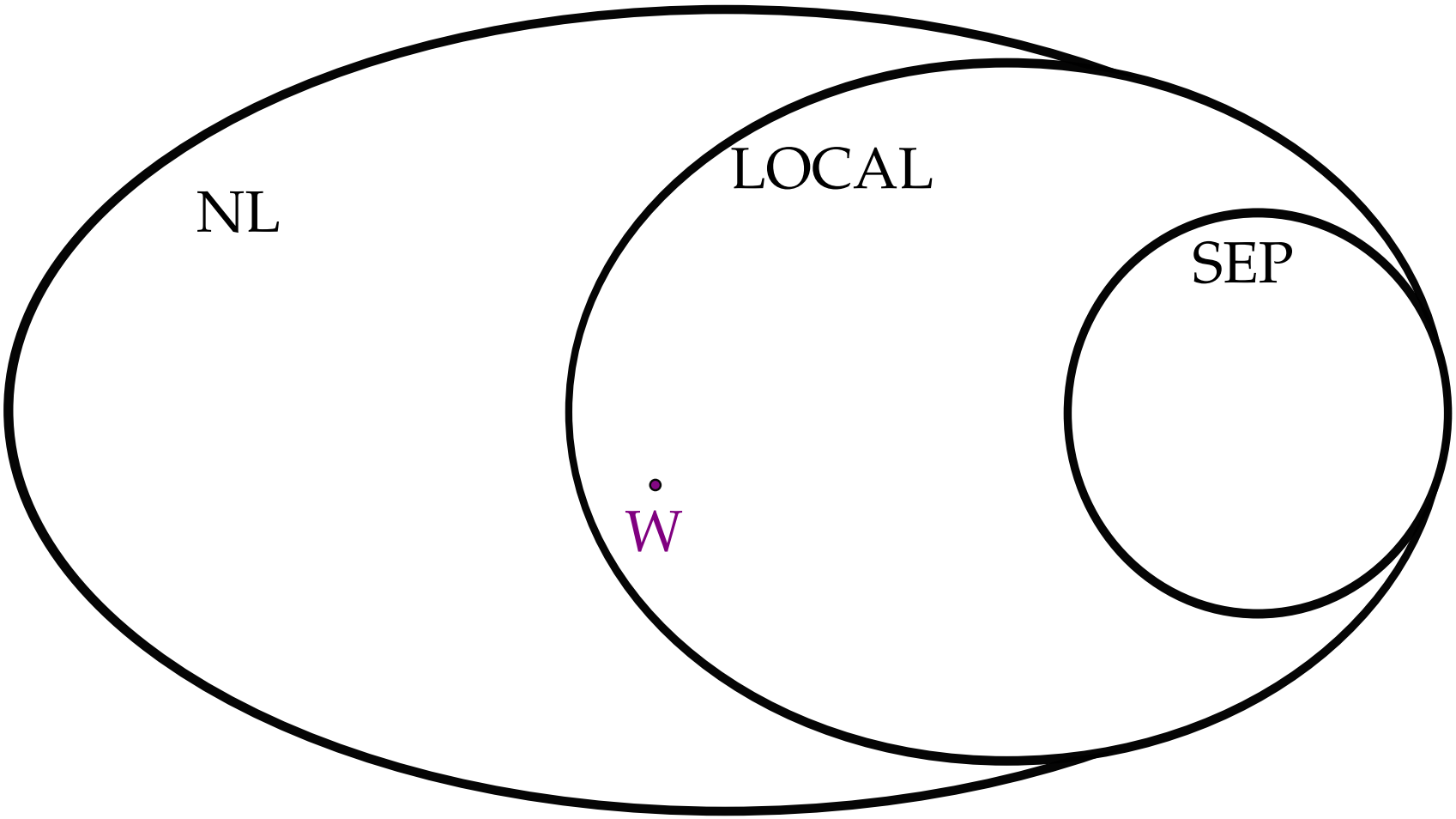
# Local simulation of quantum states

PHYSICAL REVIEW A VOLUME 40, NUMBER 8 OCTOBER 15, 1989

**Quantum states with Einstein-Podolsky-Rosen correlations admitting a hidden-variable model**

Reinhard F. Werner*
*Dublin Institute for Advanced Studies, 10 Burlington Road, Dublin 4, Ireland*
(Received 1 May 1989)

# Local simulation of quantum states



# Local simulation of quantum states

$$\lambda$$

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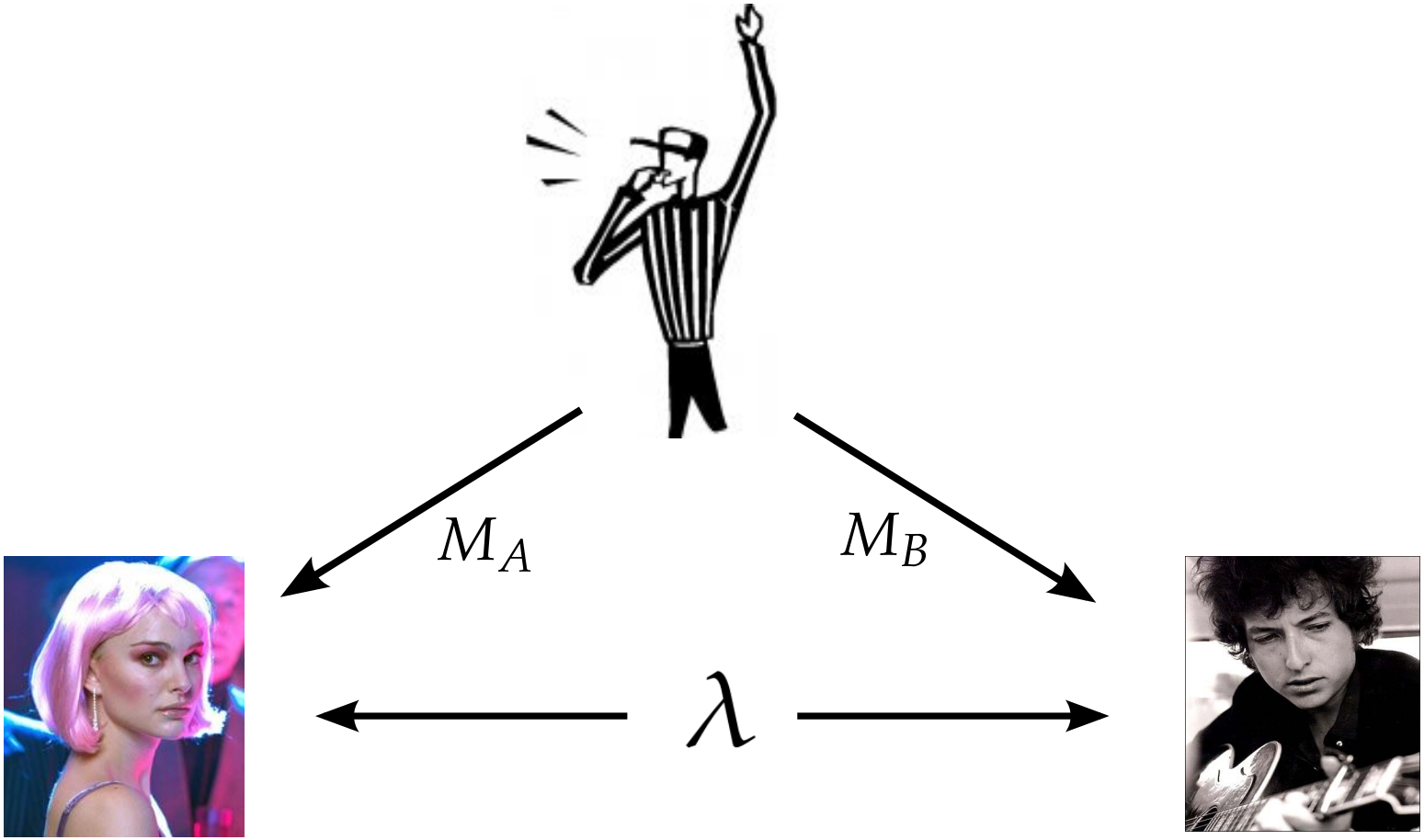

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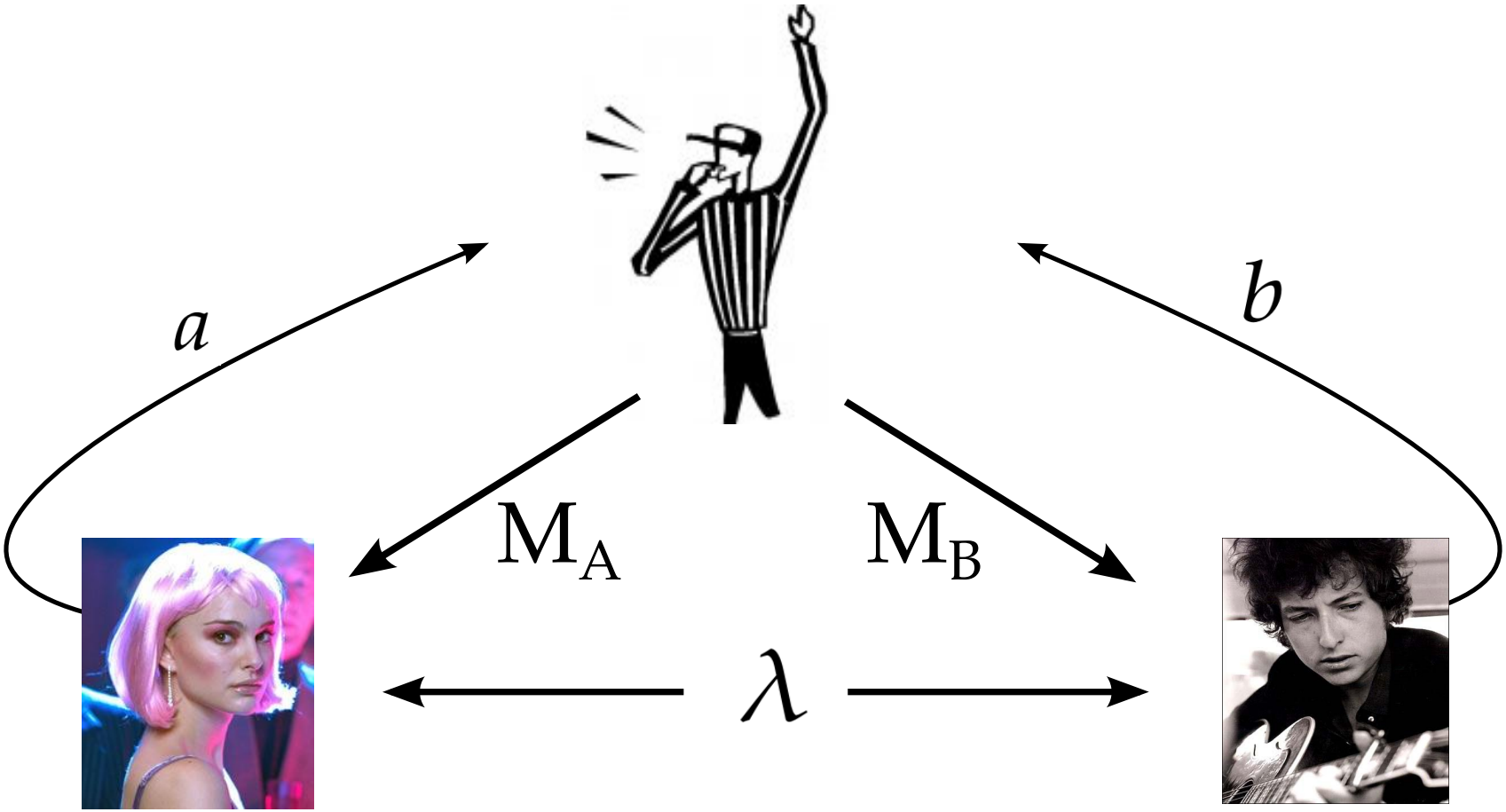

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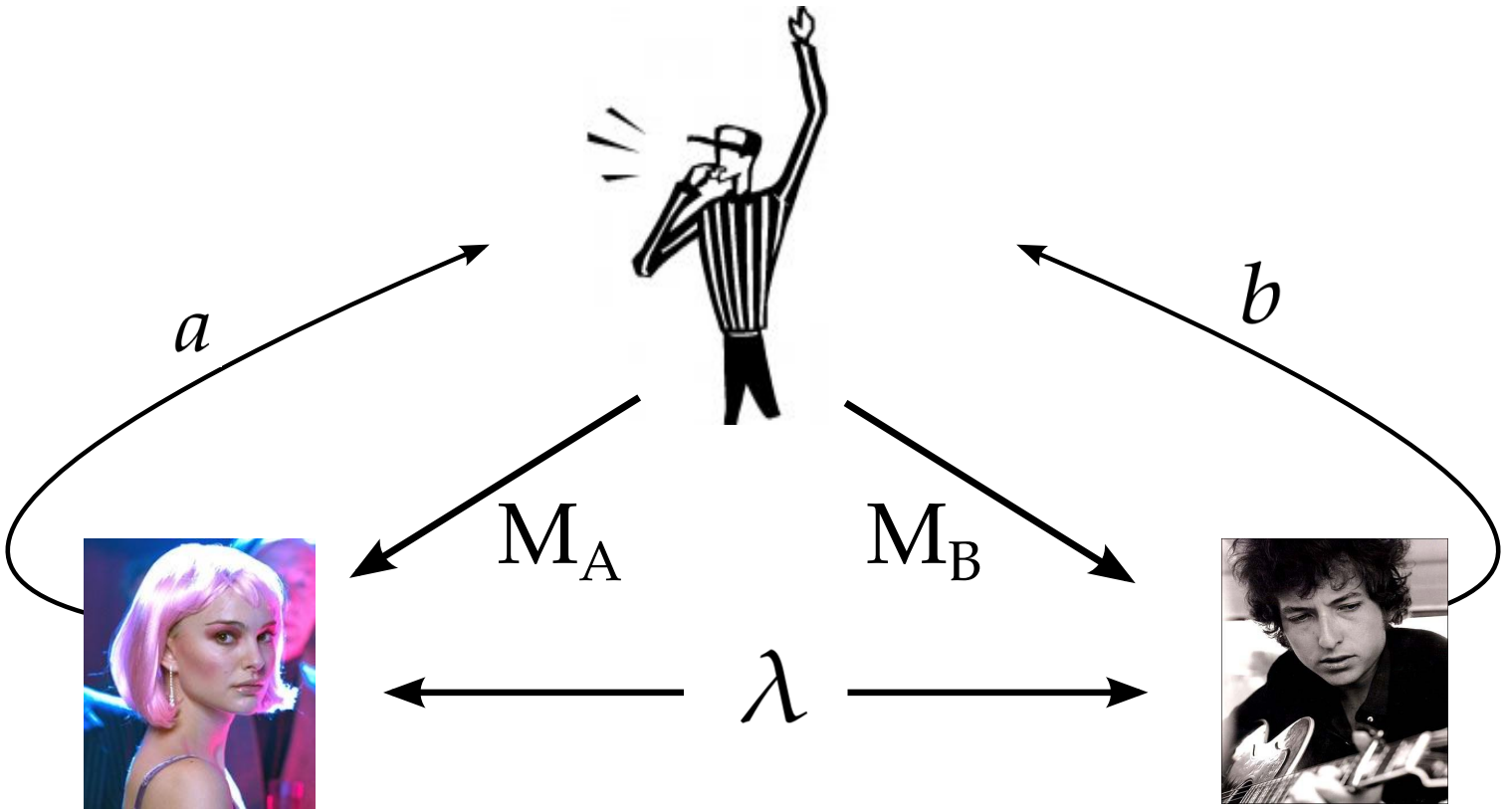
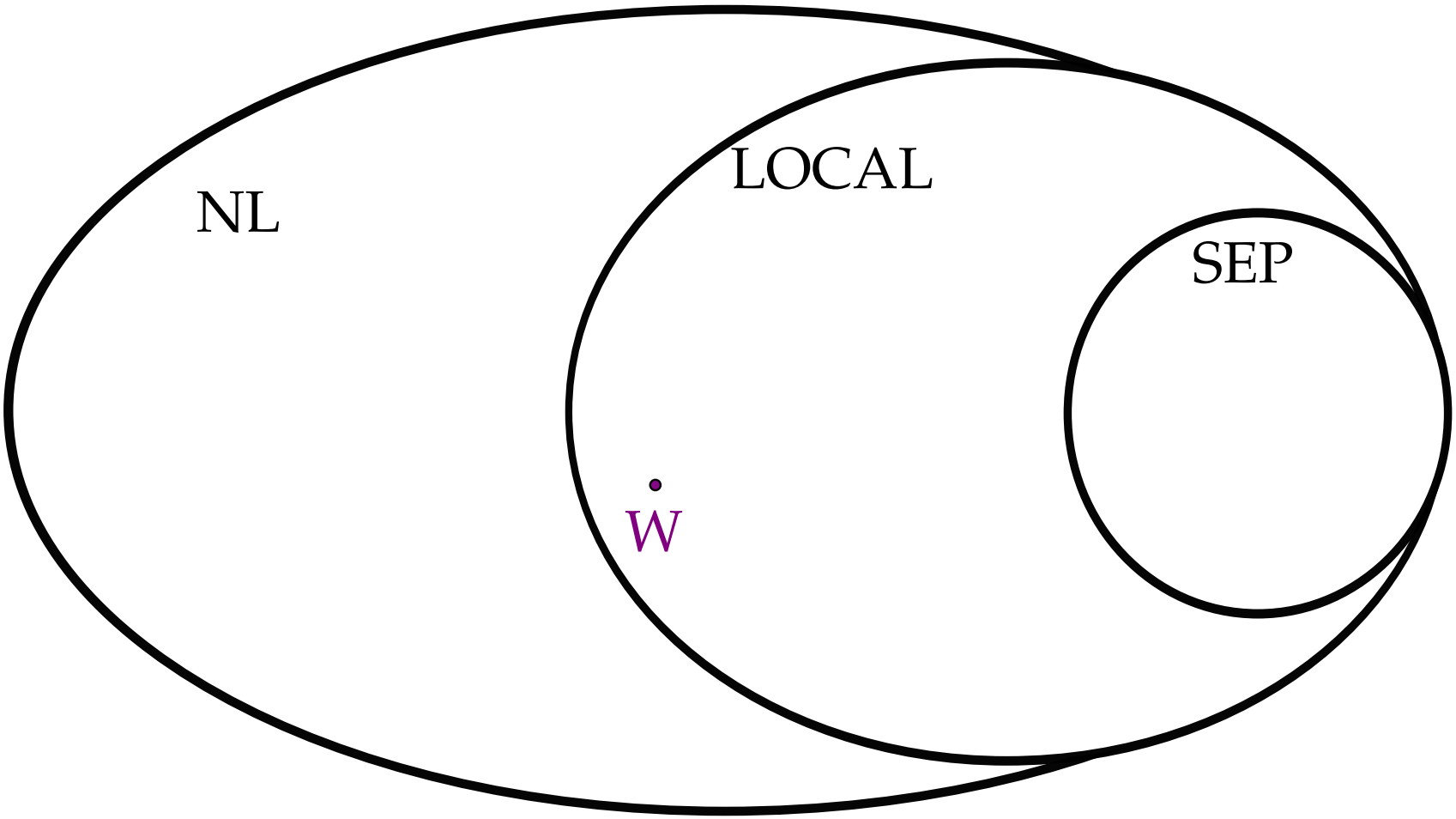



$$\int \pi(\lambda) p_A(a|\lambda) p_B(b|\lambda)\, \mathrm{d}\lambda = \mathrm{tr}(\rho_{AB} M_A^a \otimes M_B^b)$$

# Local simulation of quantum states



# End of story?

- Entanglement $\neq$ Locality!

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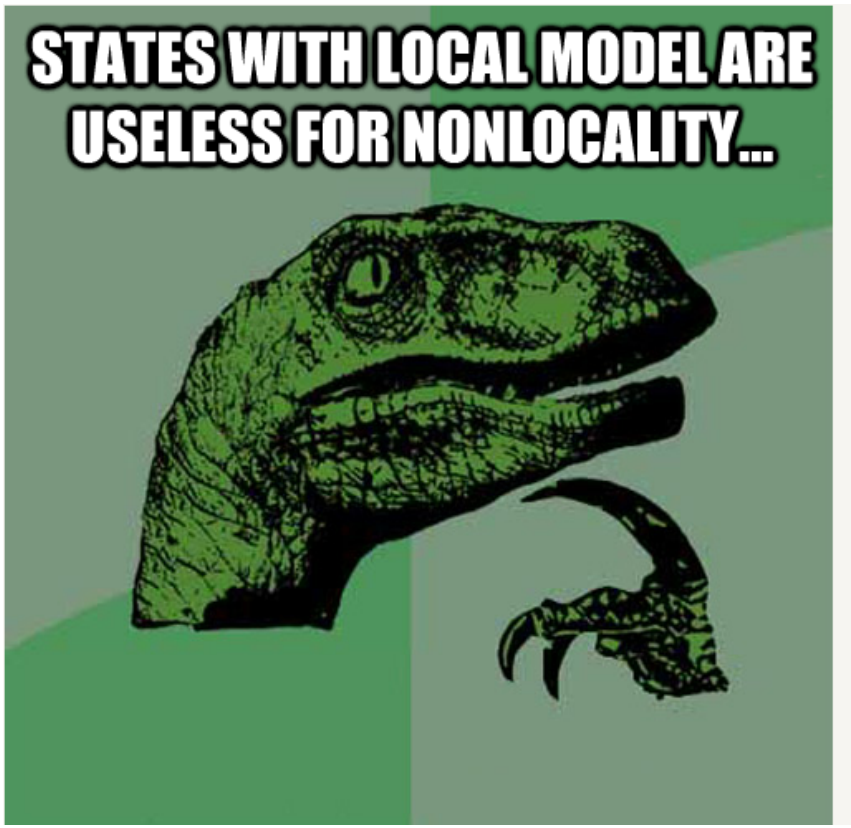
- Well...



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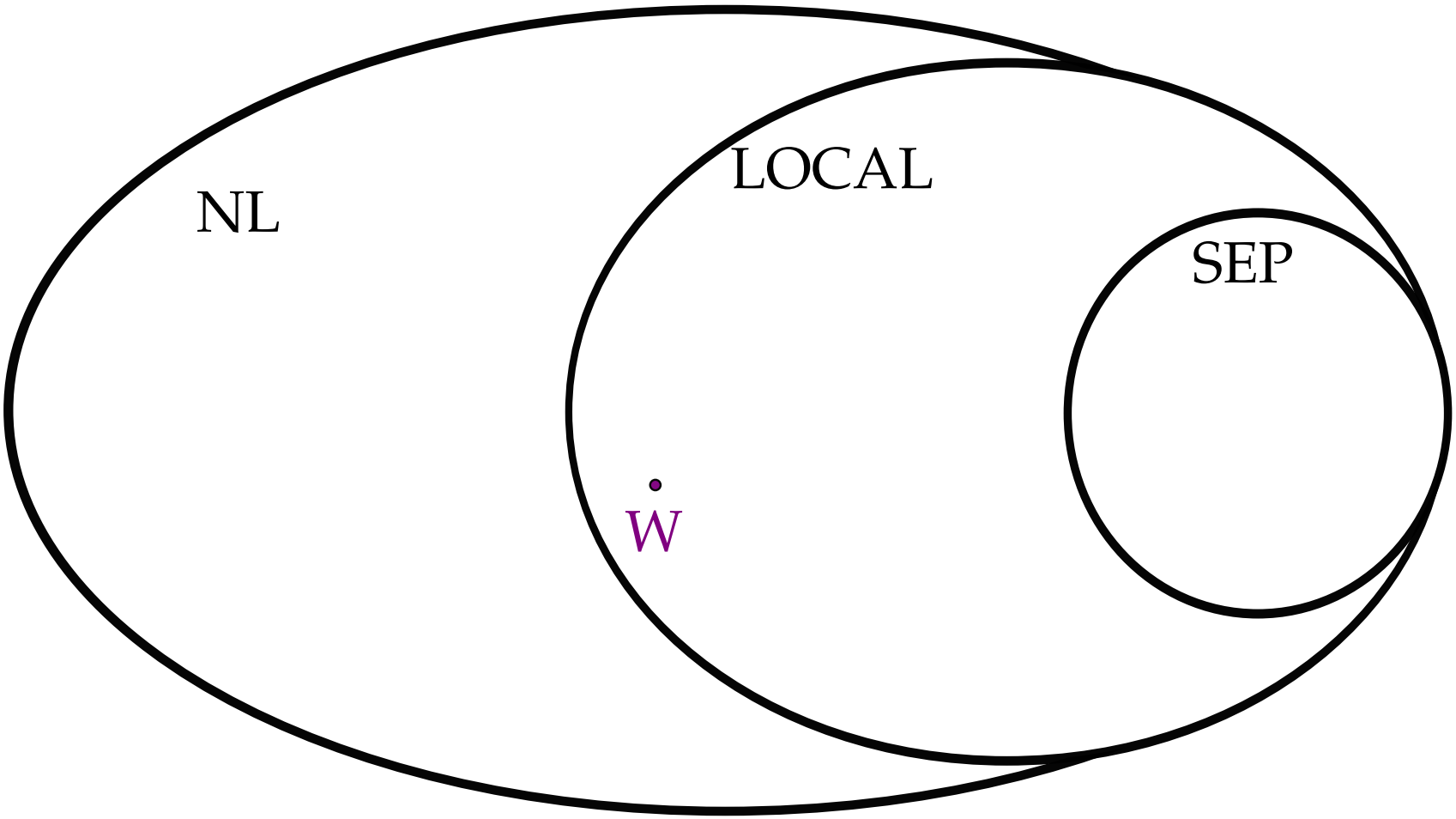
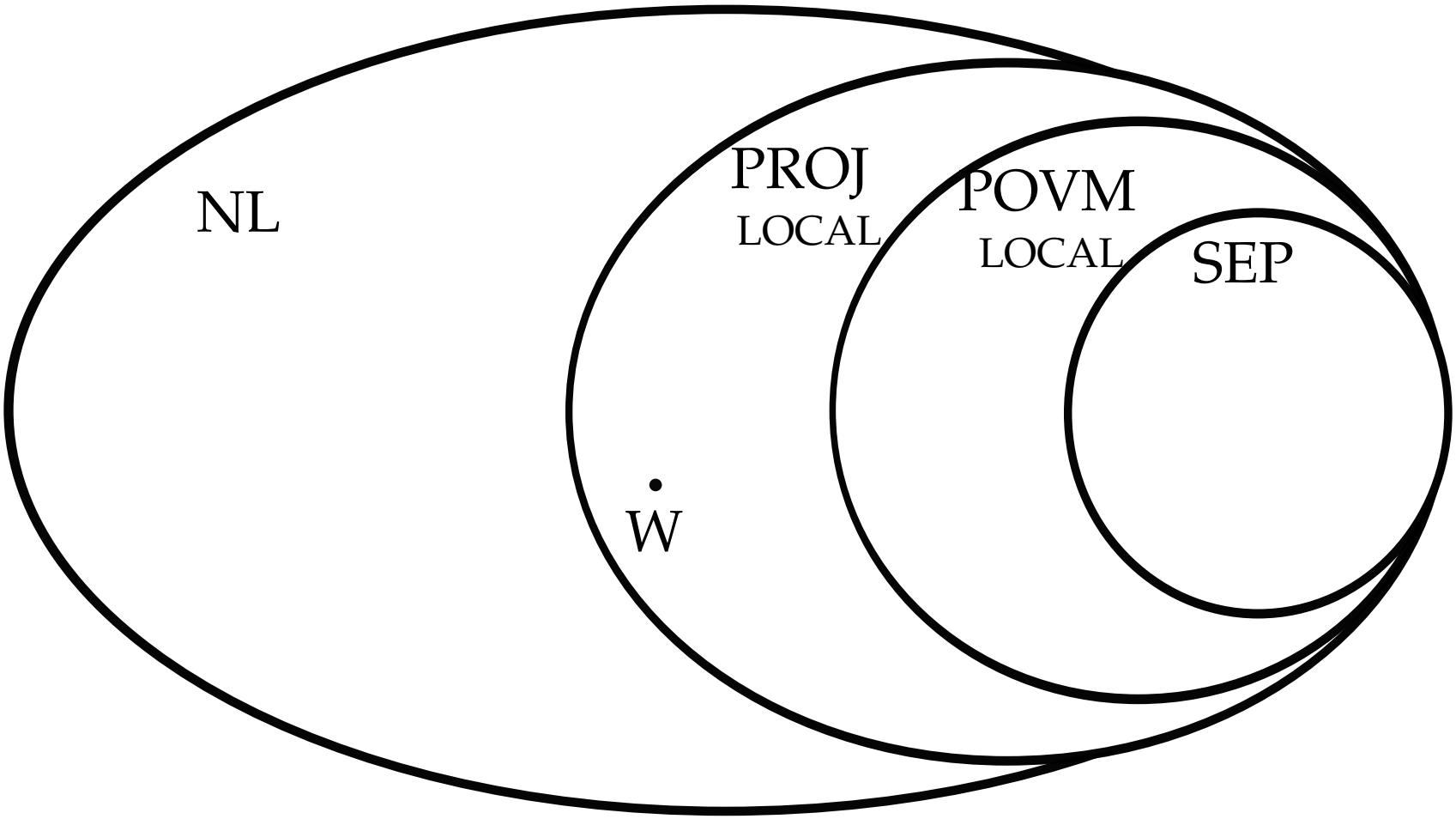


# And if we consider POVMs?



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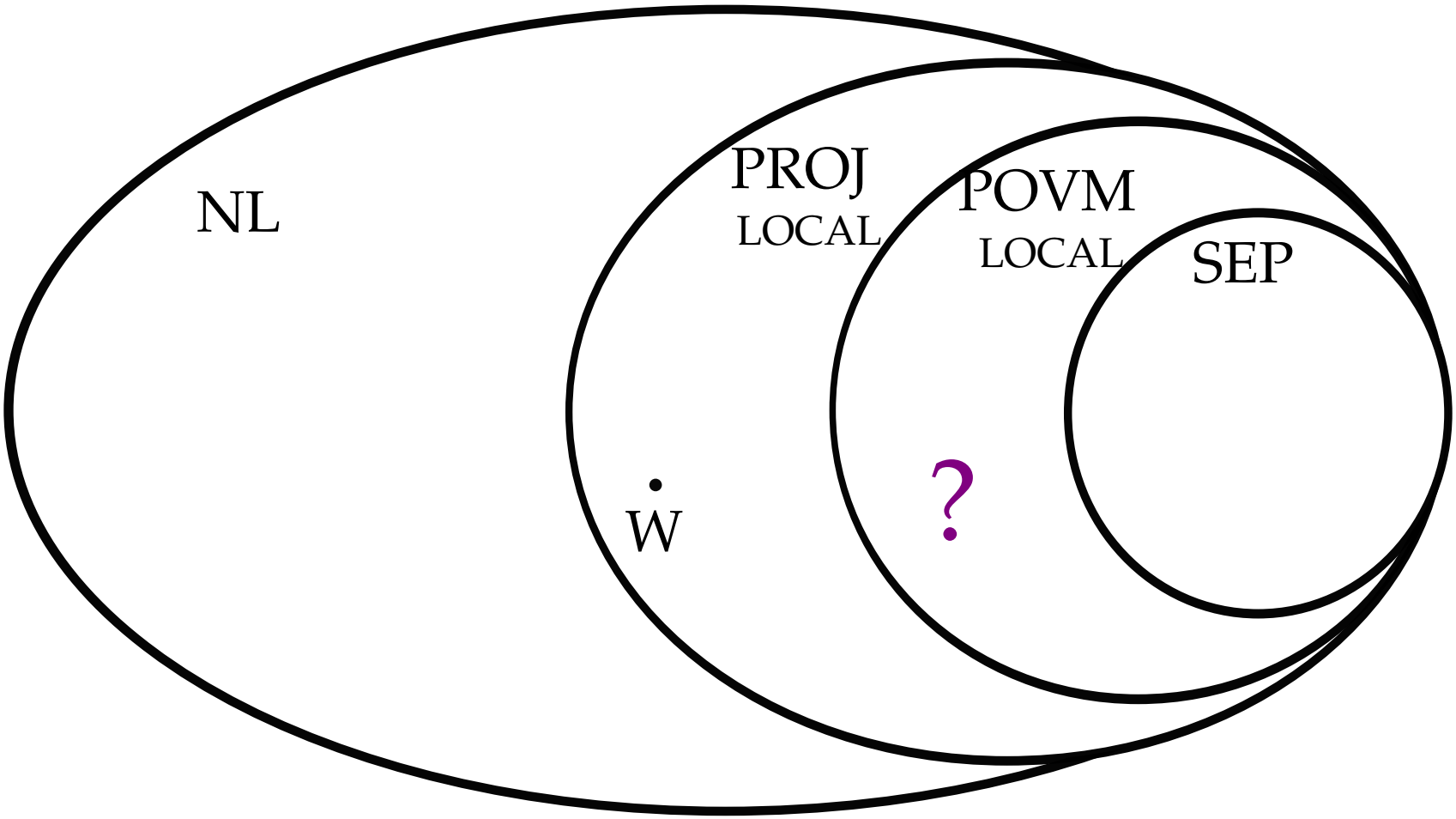
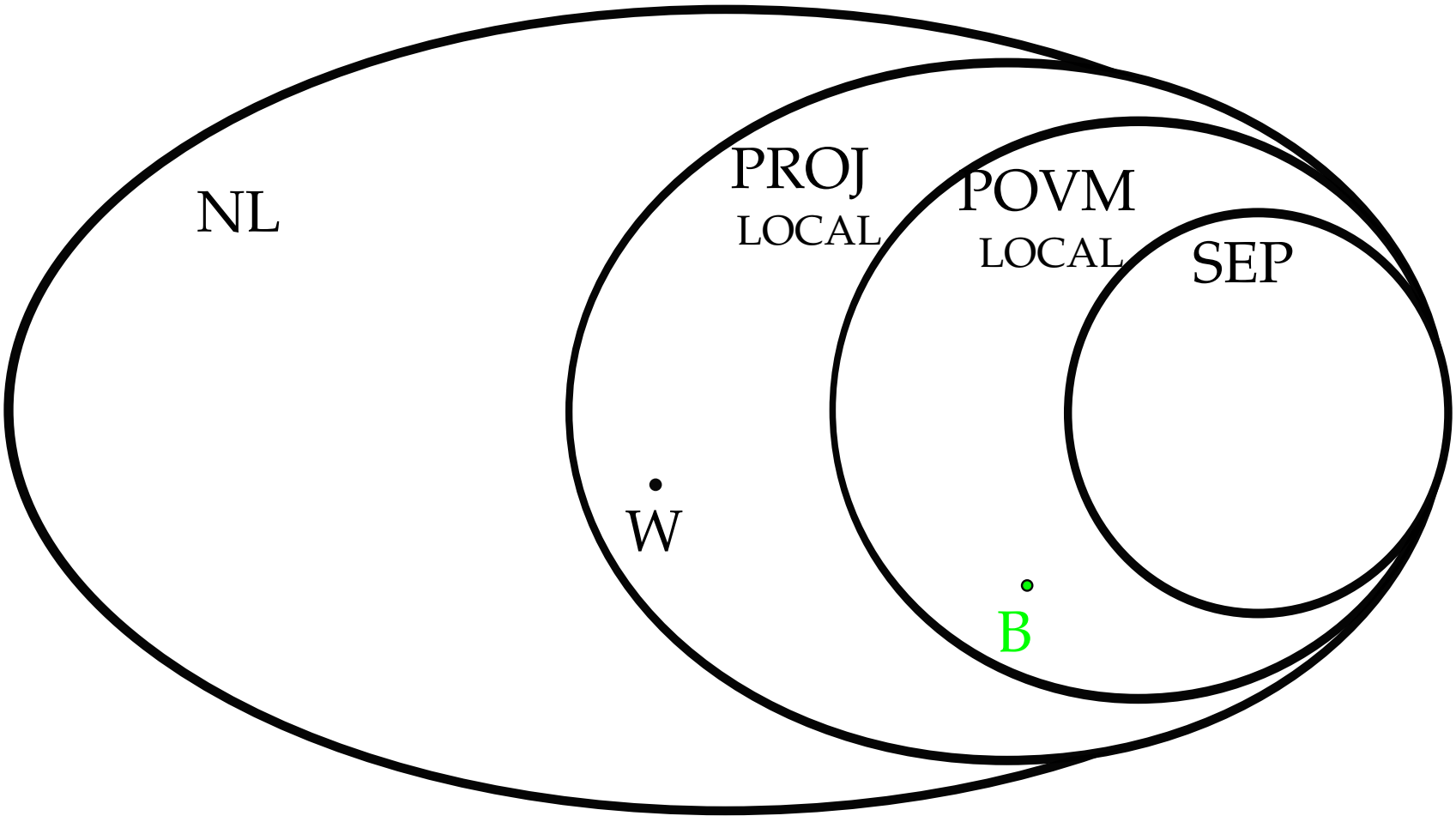
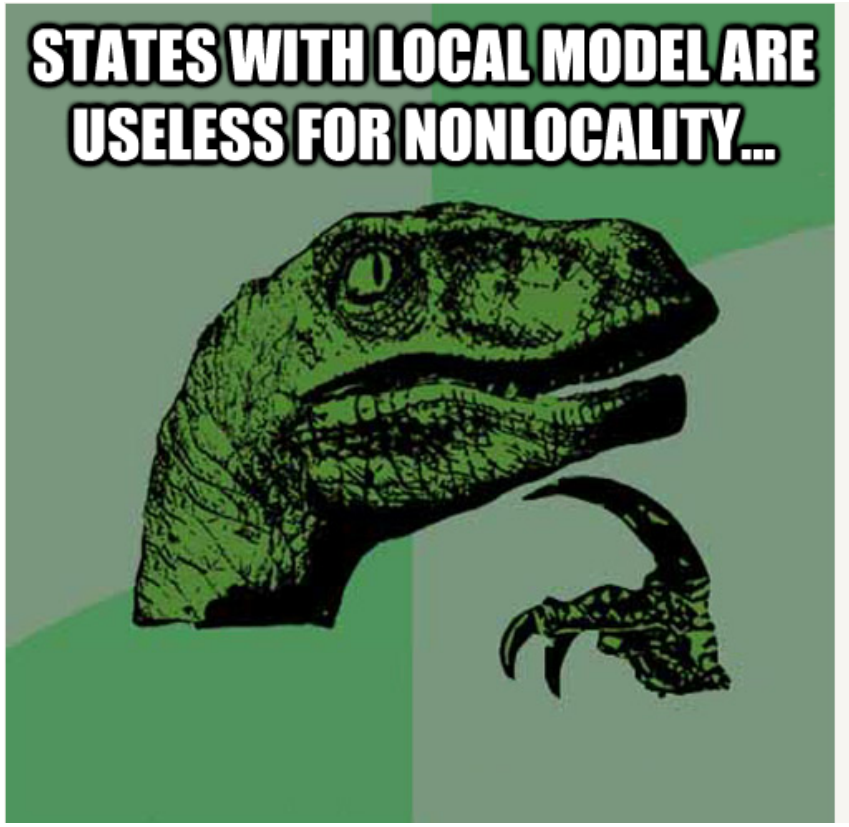


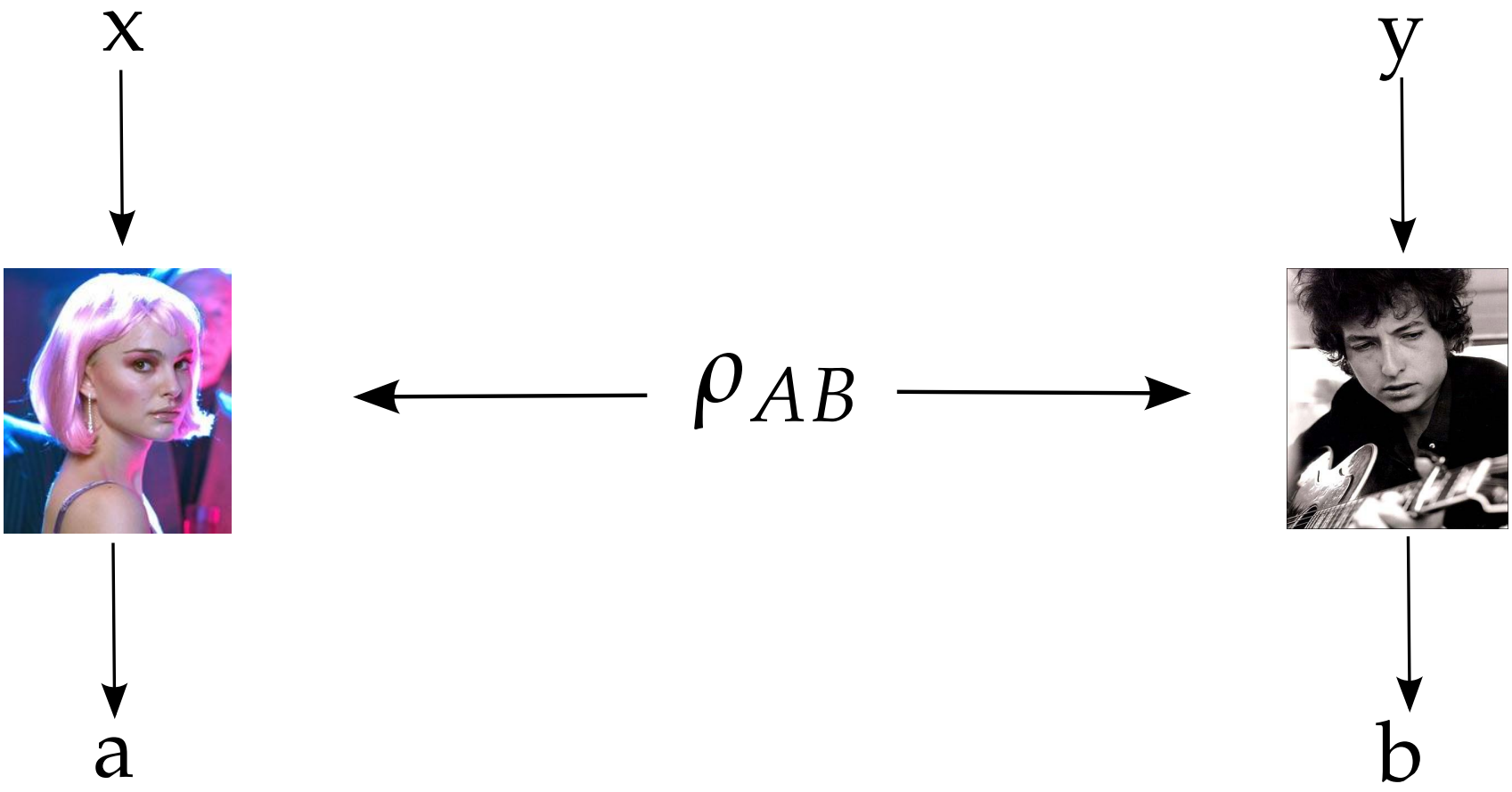
(Barrett, 02)



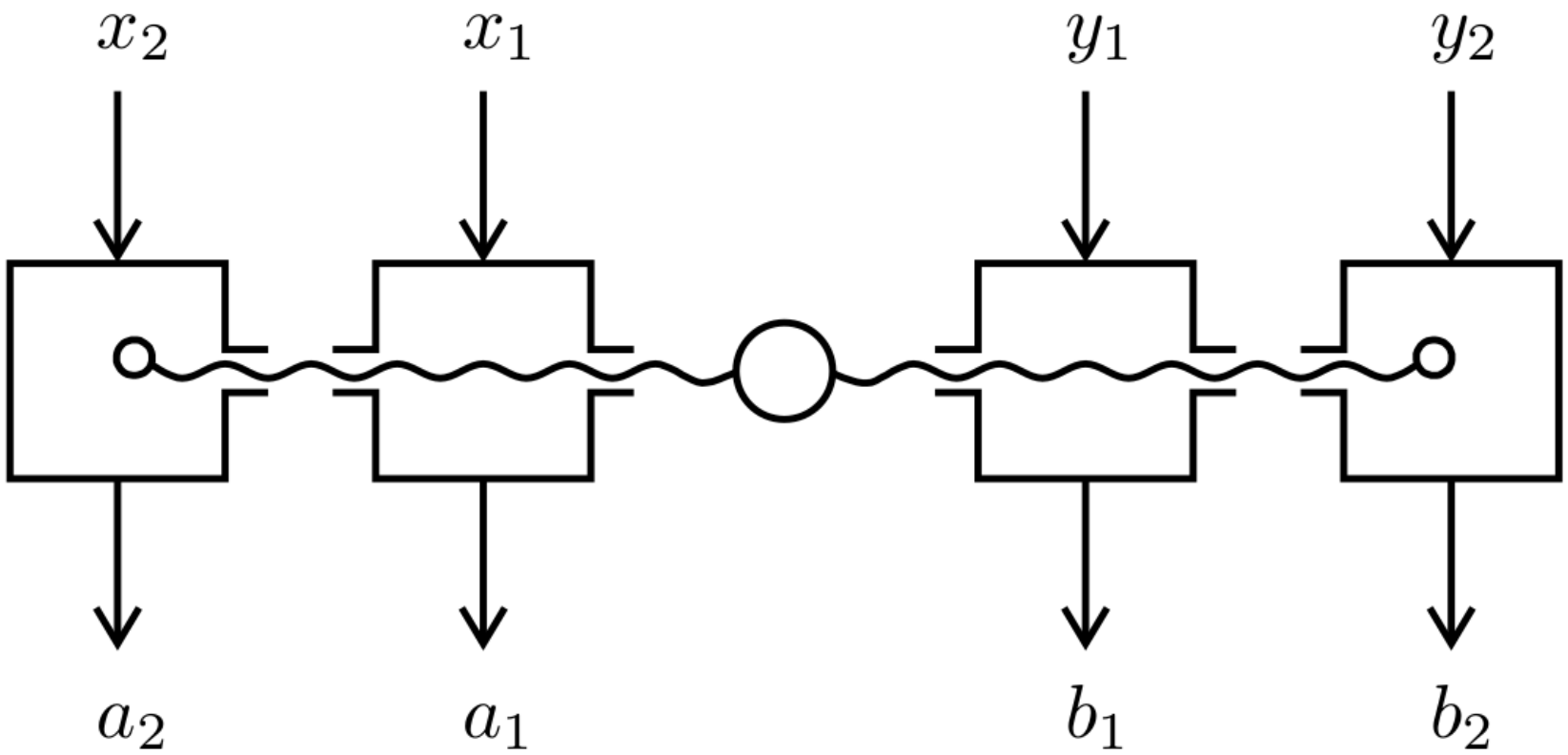
# POVM locality



# Single measurement



# Sequential measurements

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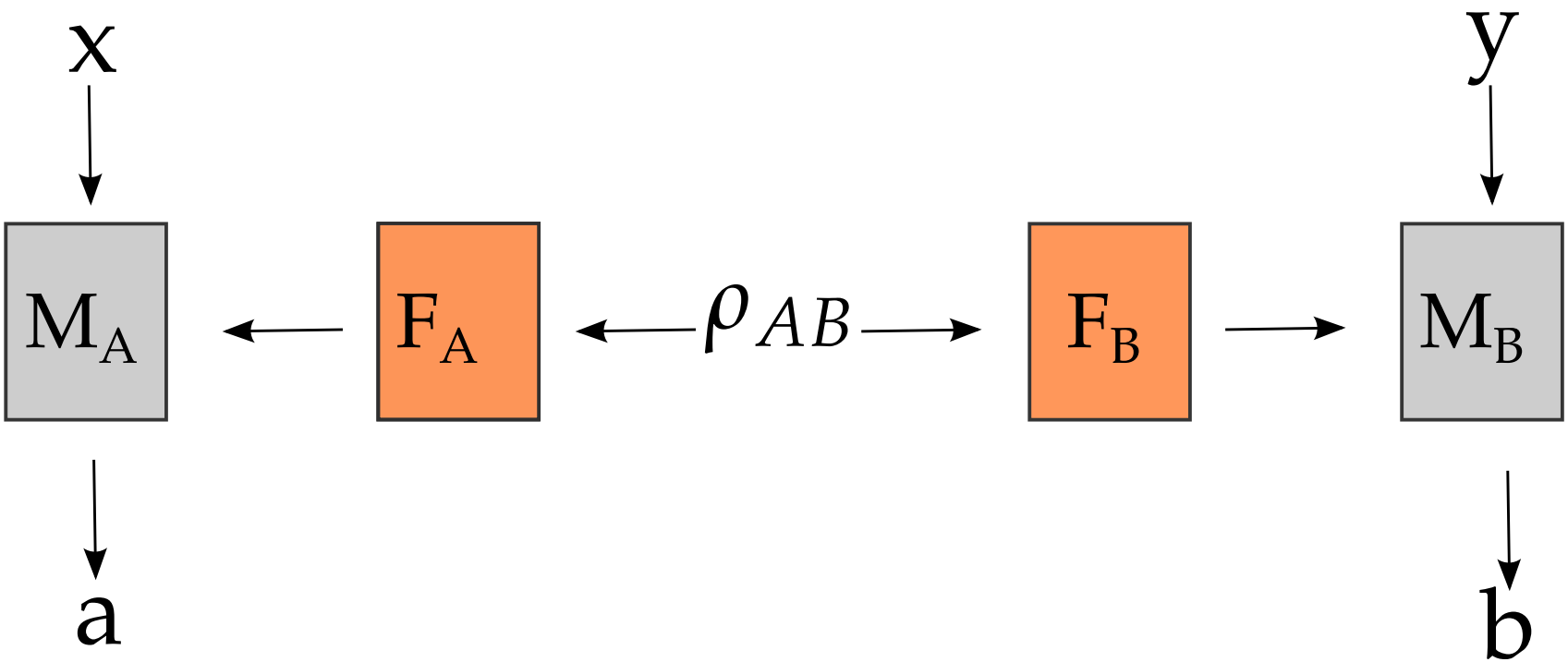

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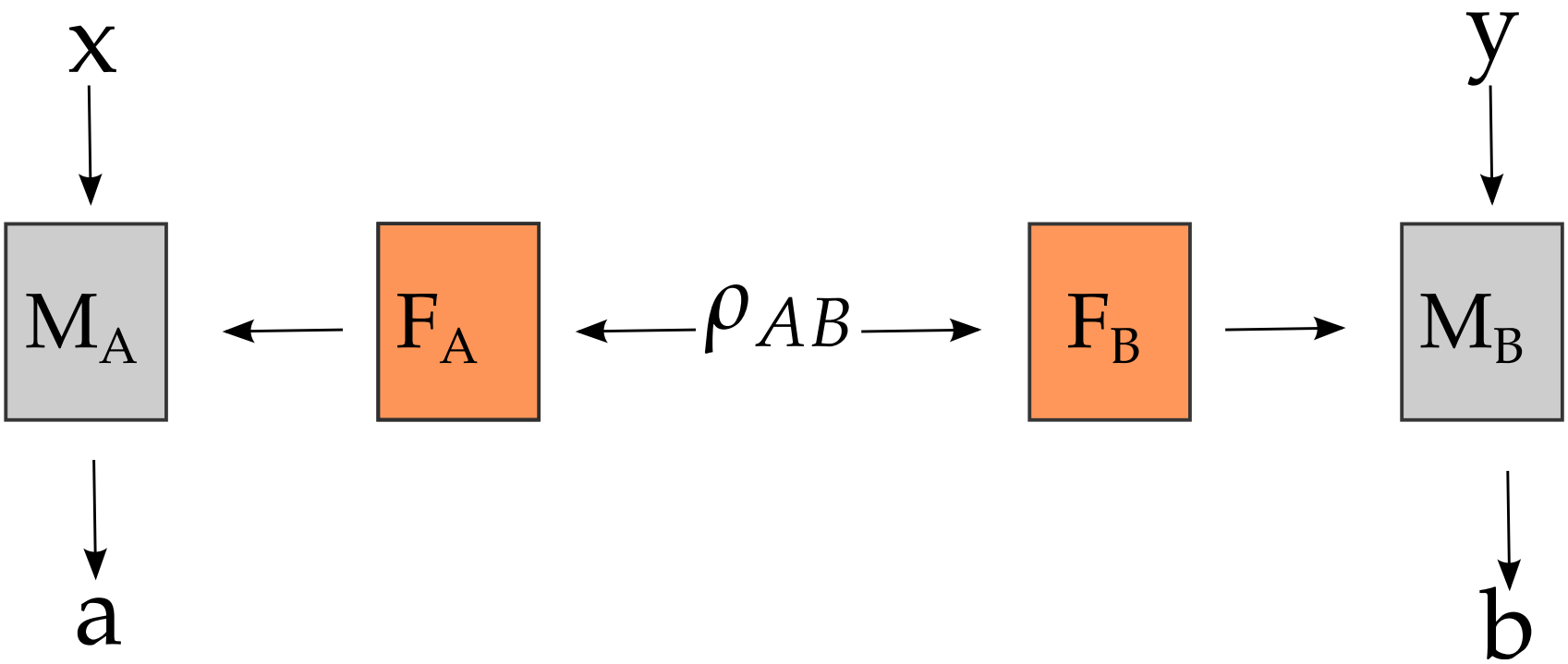
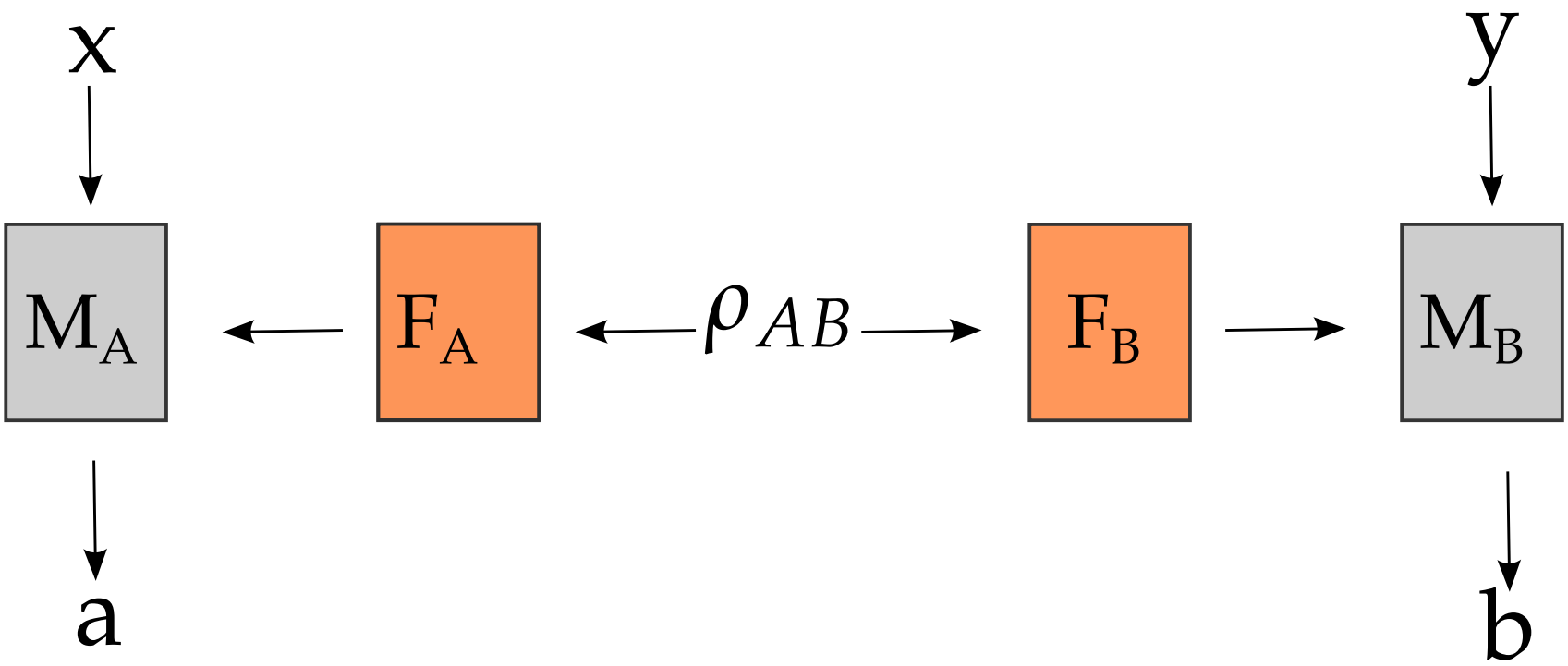



S. Popescu, Hidden Nonlocality (1995)

# No additional loophole




- Step 1 - Filtering

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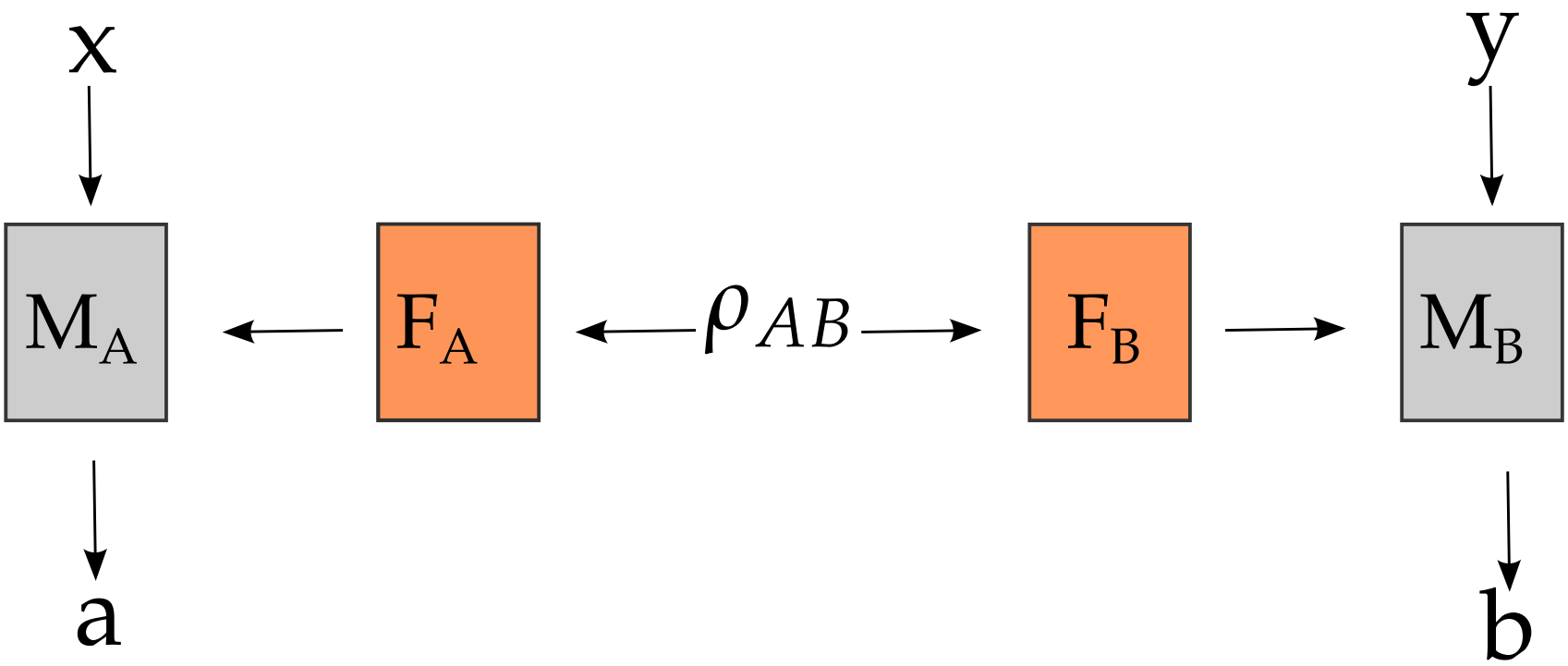


- Step 1 - Filtering
- Step 2 - Usual Bell test

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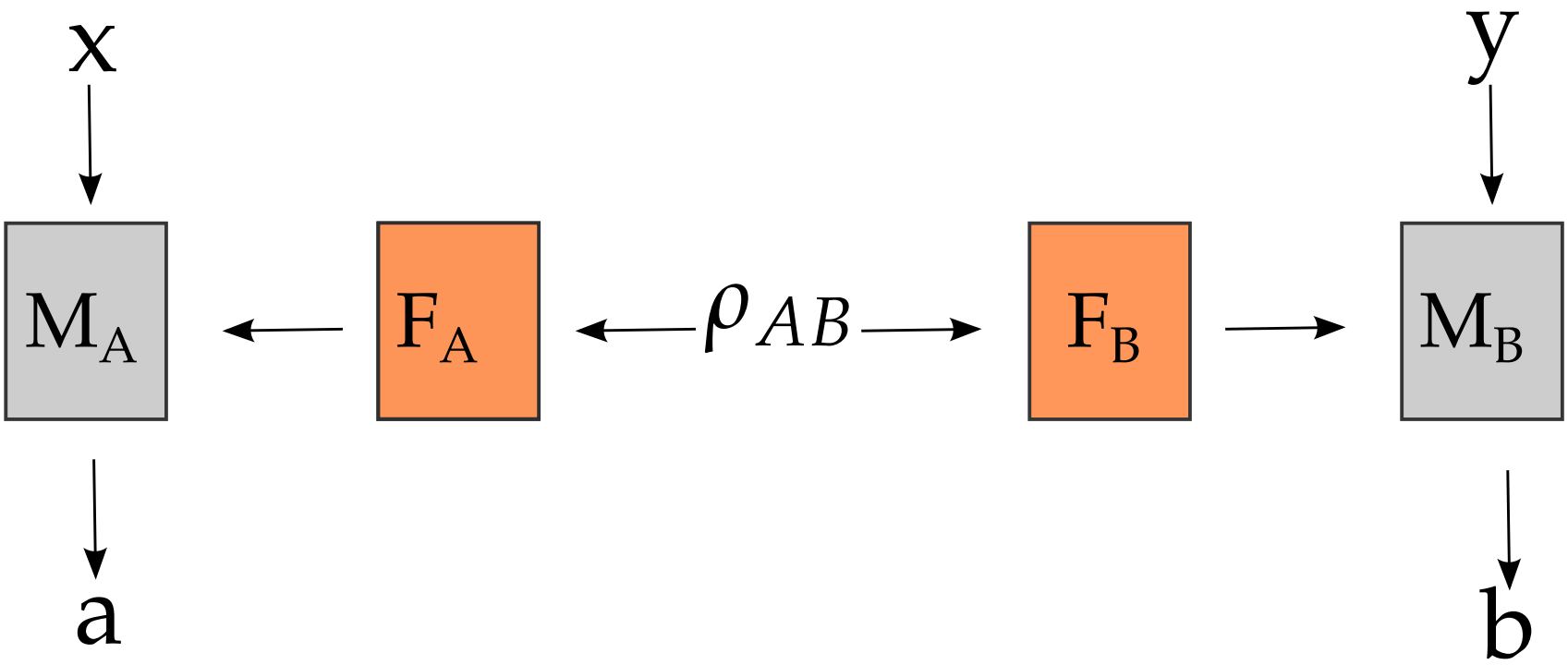


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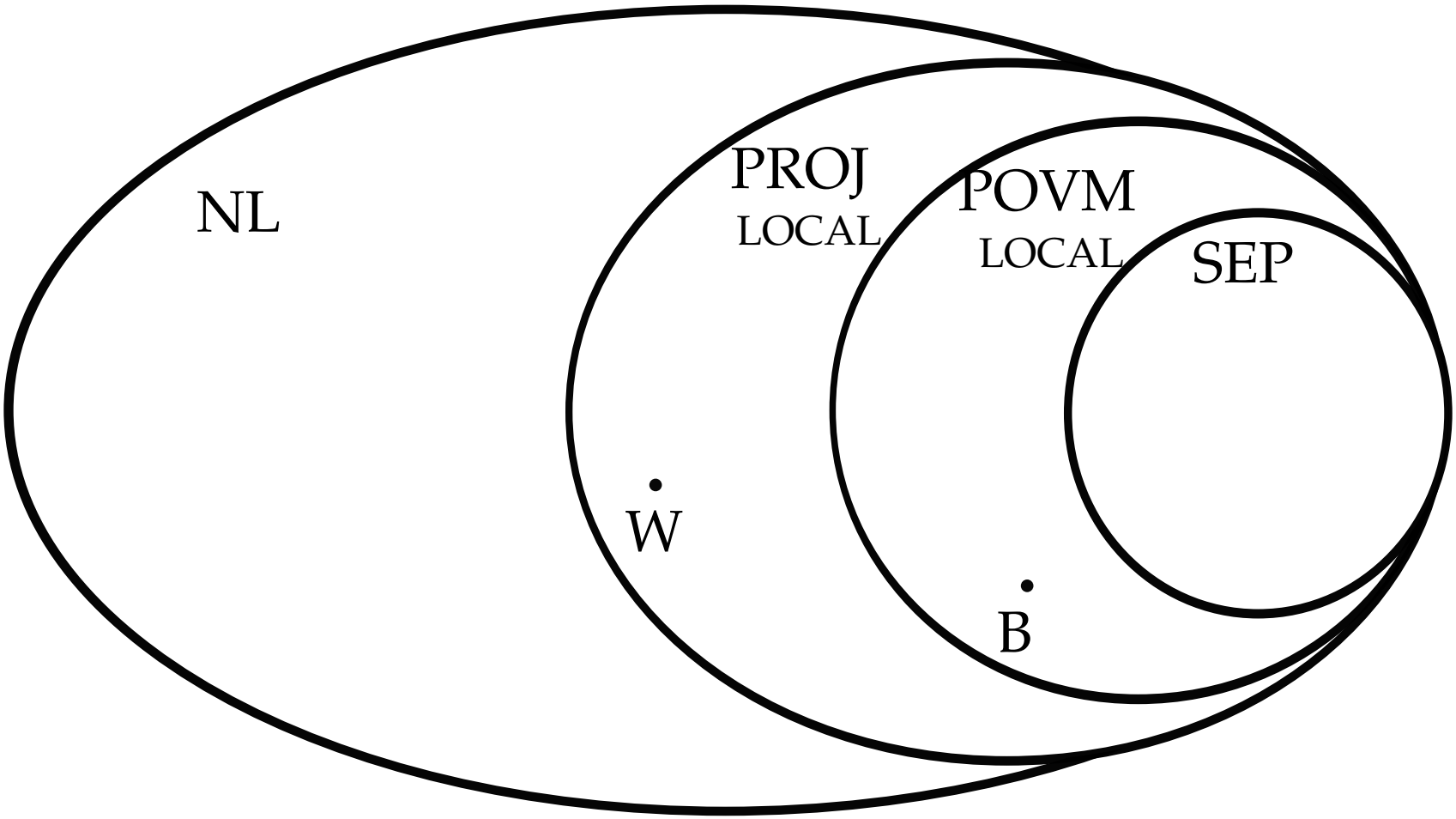
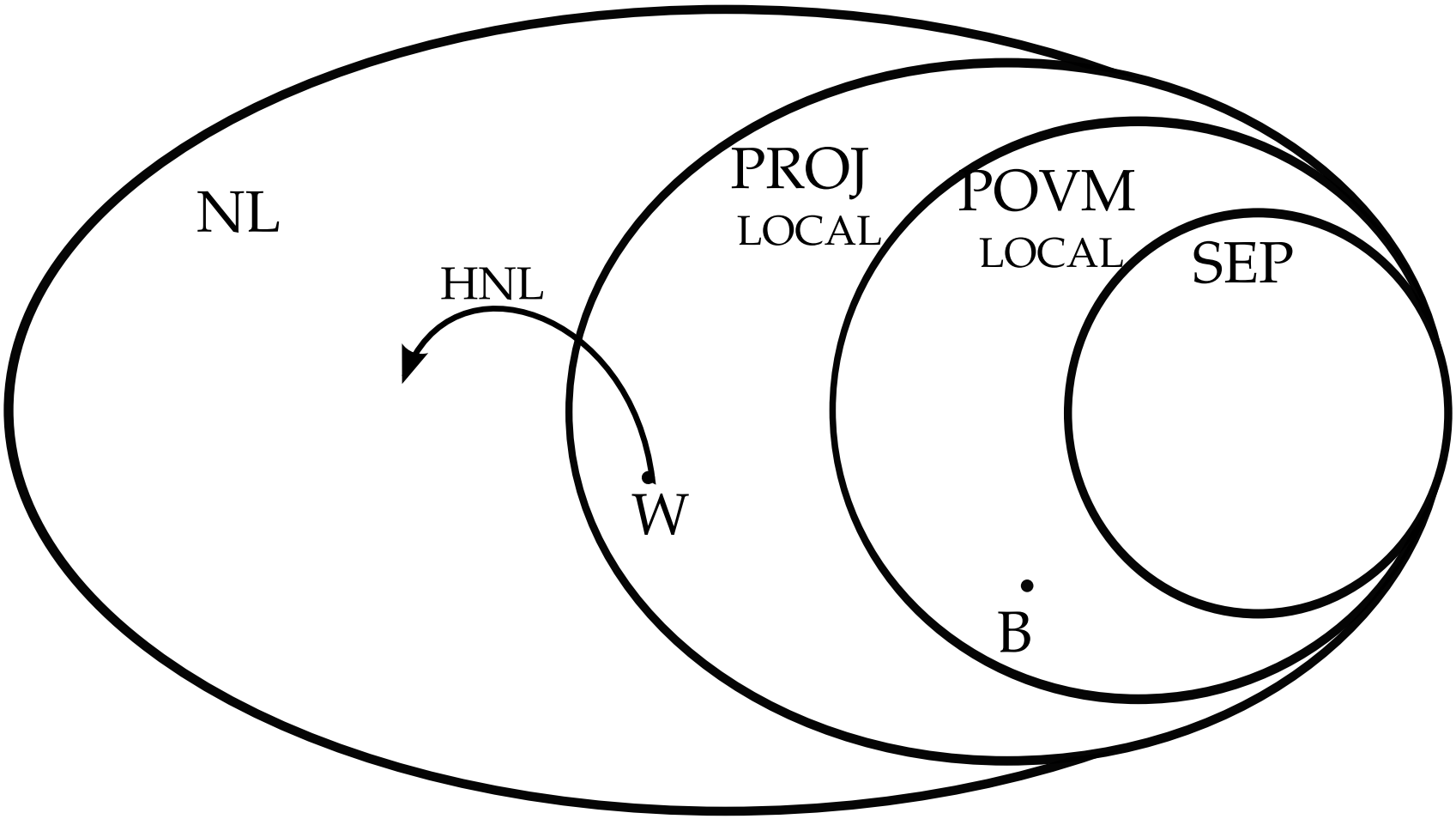
The (S)LOCC interpretation

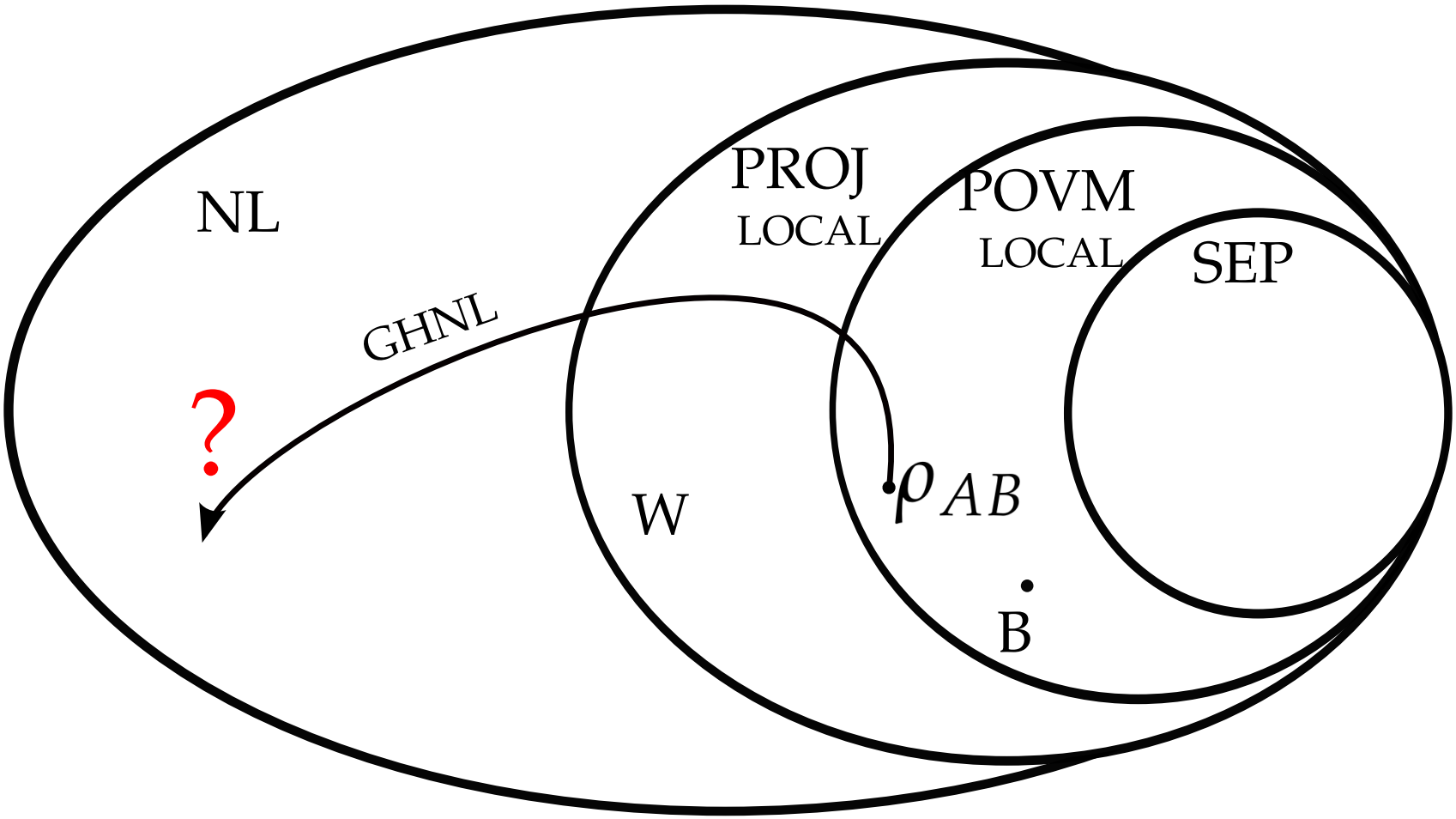
# Hidden Nonlocality



(Popescu, 95)



# Genuine Hidden Nonlocality?



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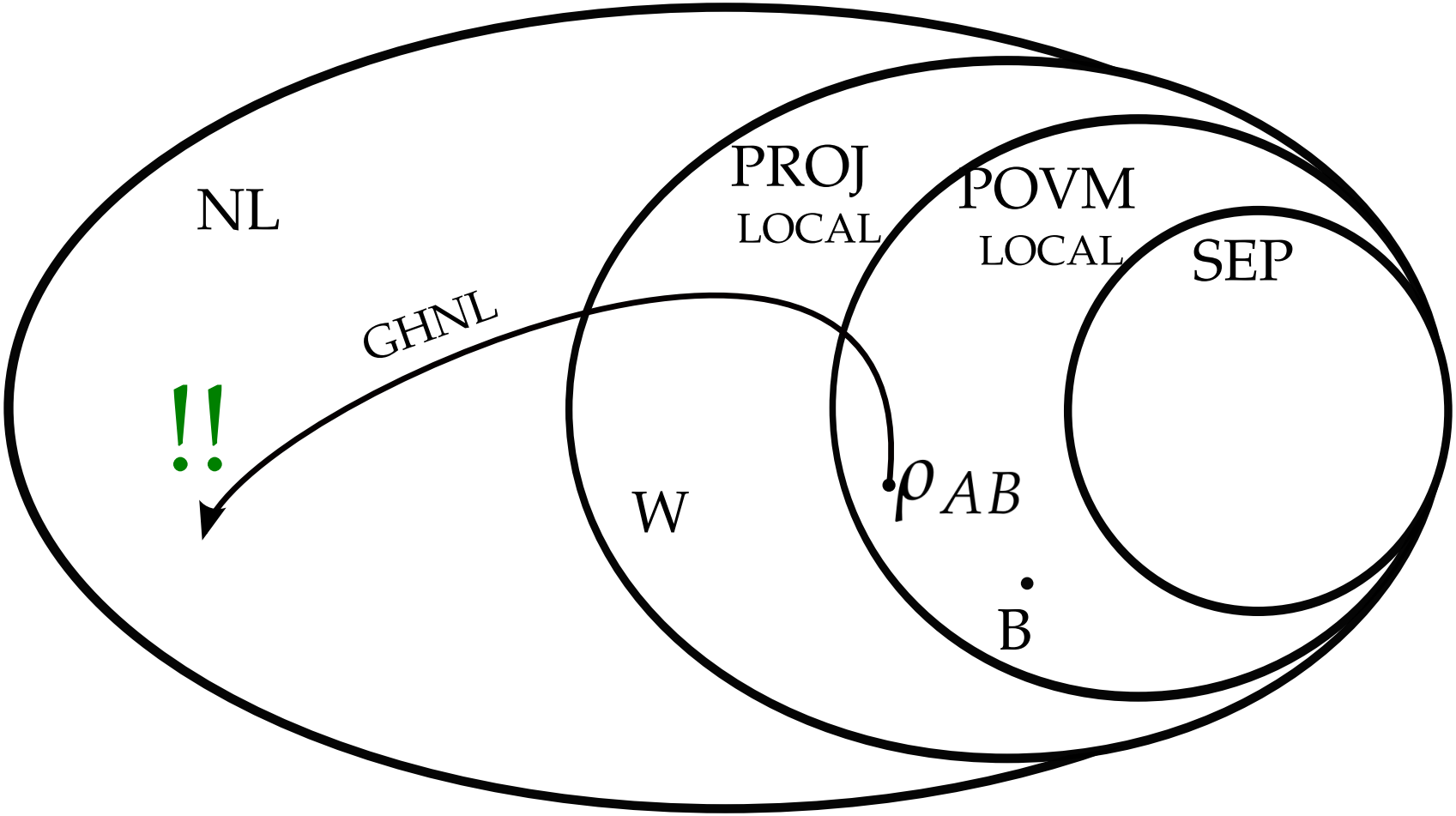
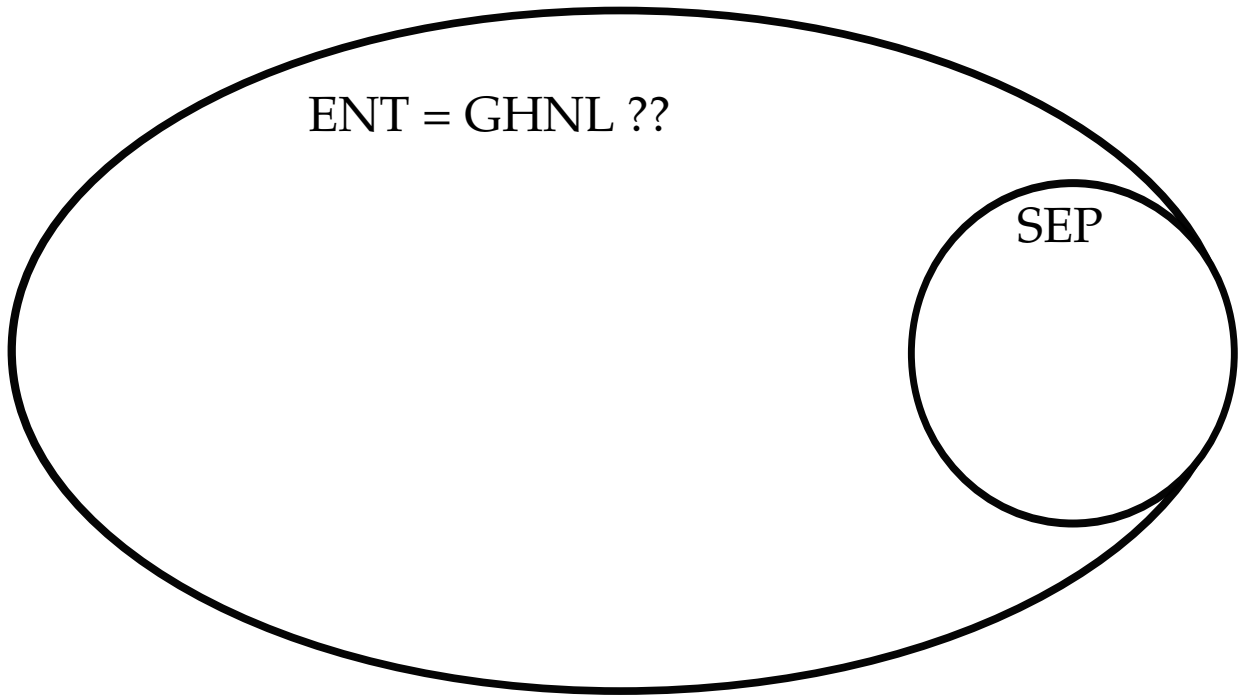



Genuine hidden quantum nonlocality
F. Hirsch, M.T. Quintino, J. Bowles, N. Brunner
Phys. Rev. Lett. 111, 160402 (2013)

# Our contribution

- Hidden nonlocal $\Longleftrightarrow$ entangled ?



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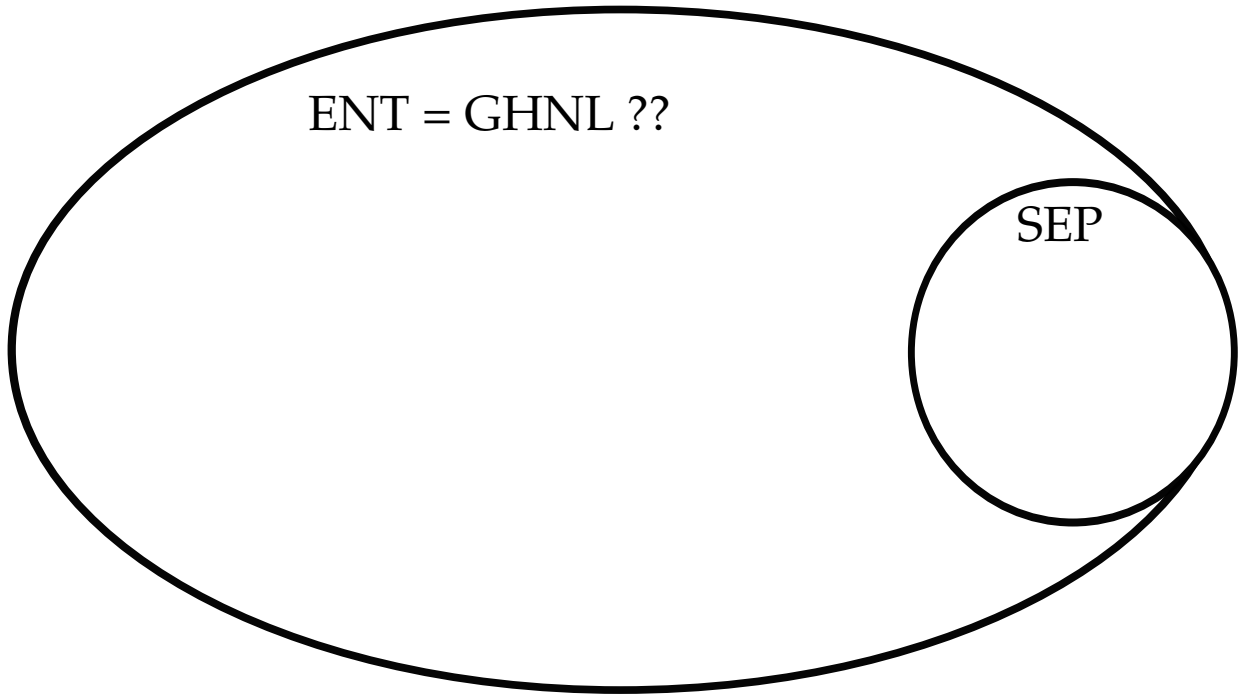


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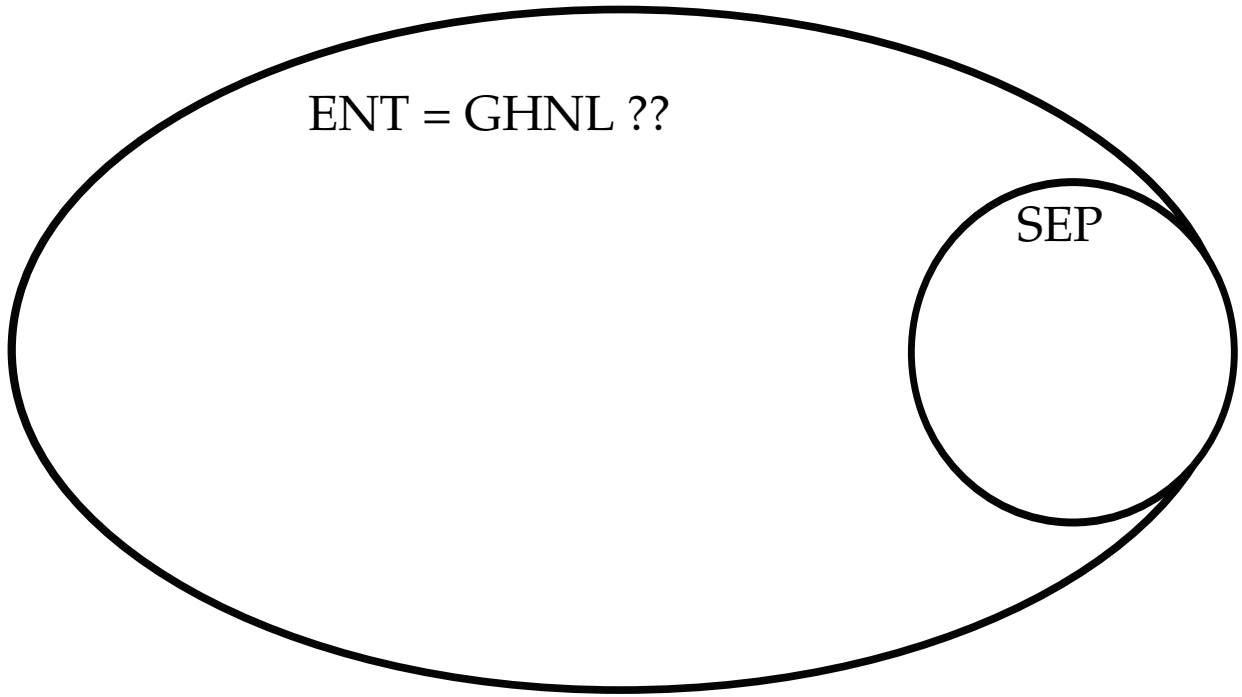


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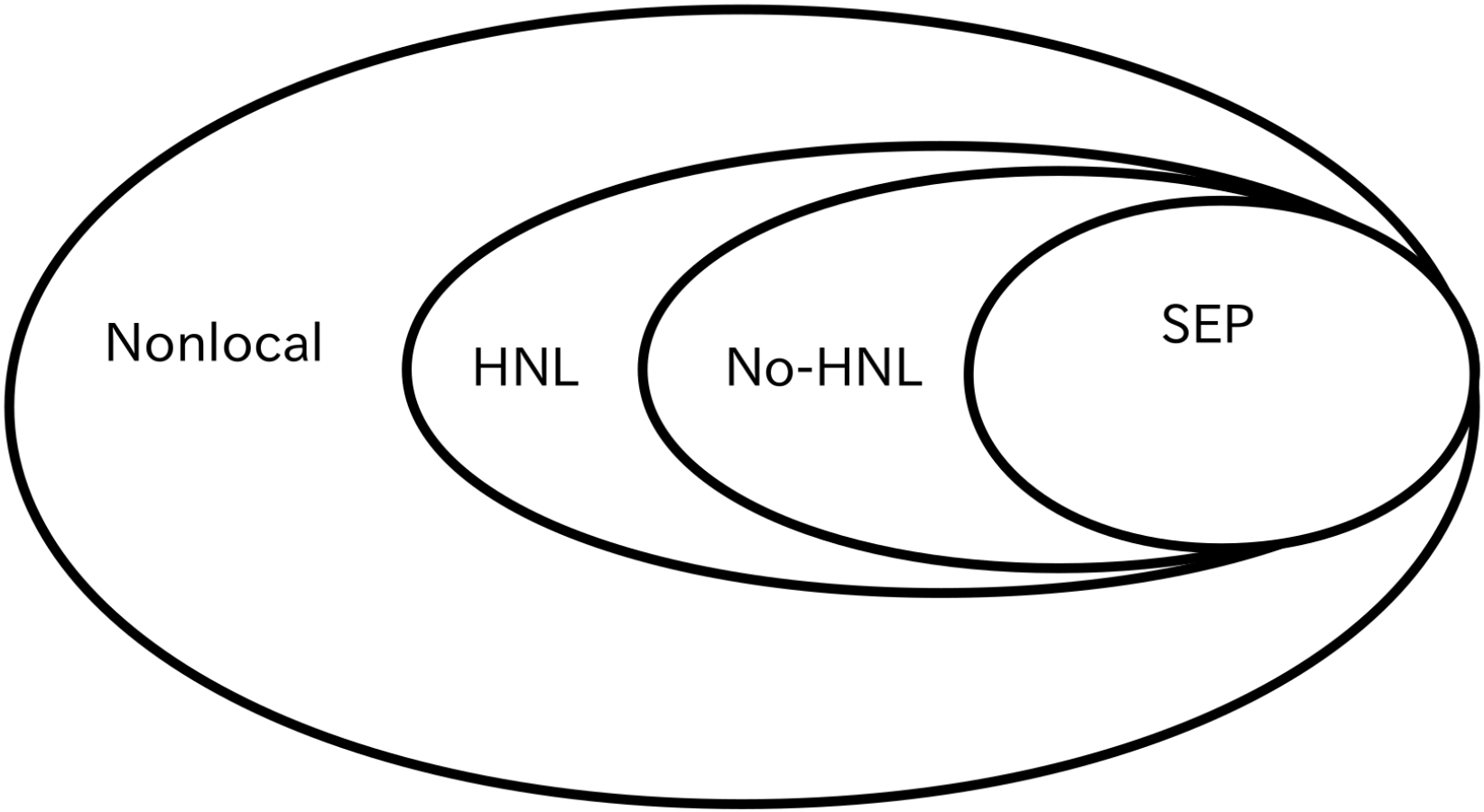
# Our contribution



# Precise result

**Theorem**

*For some visibility $V$ the two qubit Werner state*

$$W = V\psi^- + (1 - V)\frac{I}{4}$$

*is entangled and does not have hidden nonlocality.*
*PROJ:* $V \leq 1/2$
*POVMs:* $V \leq 0.4$

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- NL cannot be activated by SLOCC
- NL cannot be activated by LOCC

"Hey Marco, write something in the board!"

# Methods!

Step 1 - Equivalent resources for NL on entangled states:

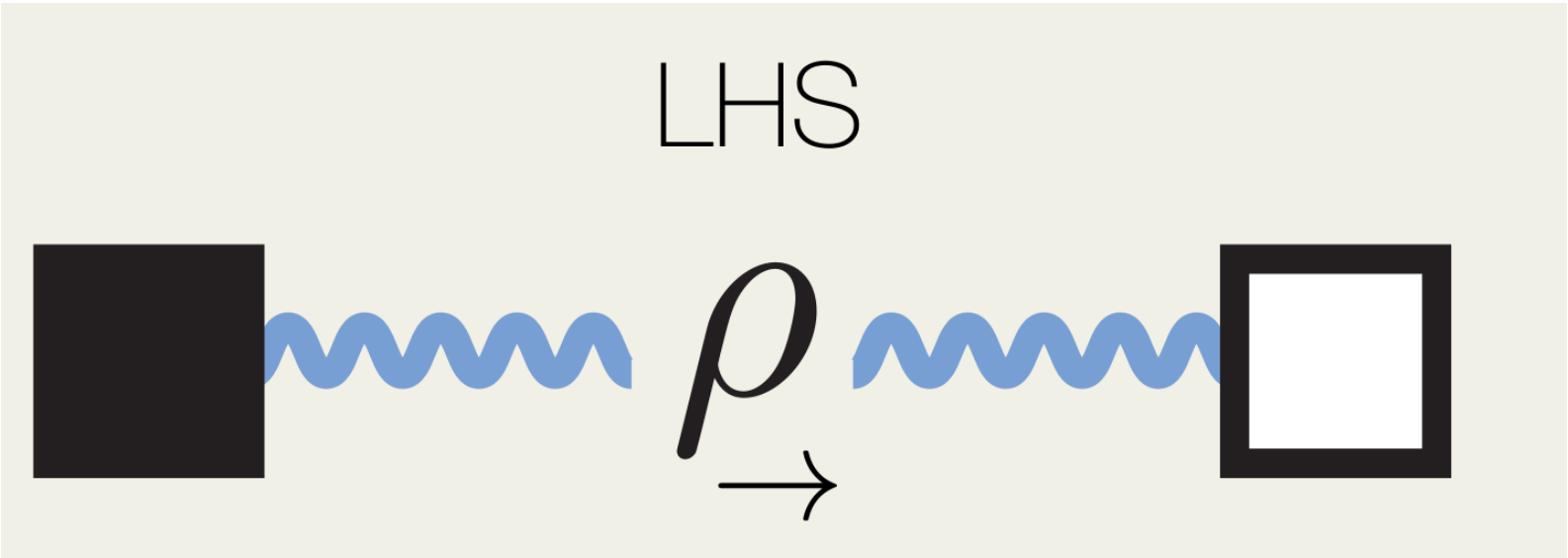
- No sequential NL for single choice in the first round
- NL cannot be activated by local filtering
- NL cannot be activated by SLOCC
- NL cannot be activated by LOCC

Asymptotic violation of Bell inequalities and distillability
Ll. Masanes
Phys. Rev. Lett. 97, 050503 (2006)

# Methods!

Step 2 - General results for local models

- Steering local models "respect" quantum mechanics in characterised part

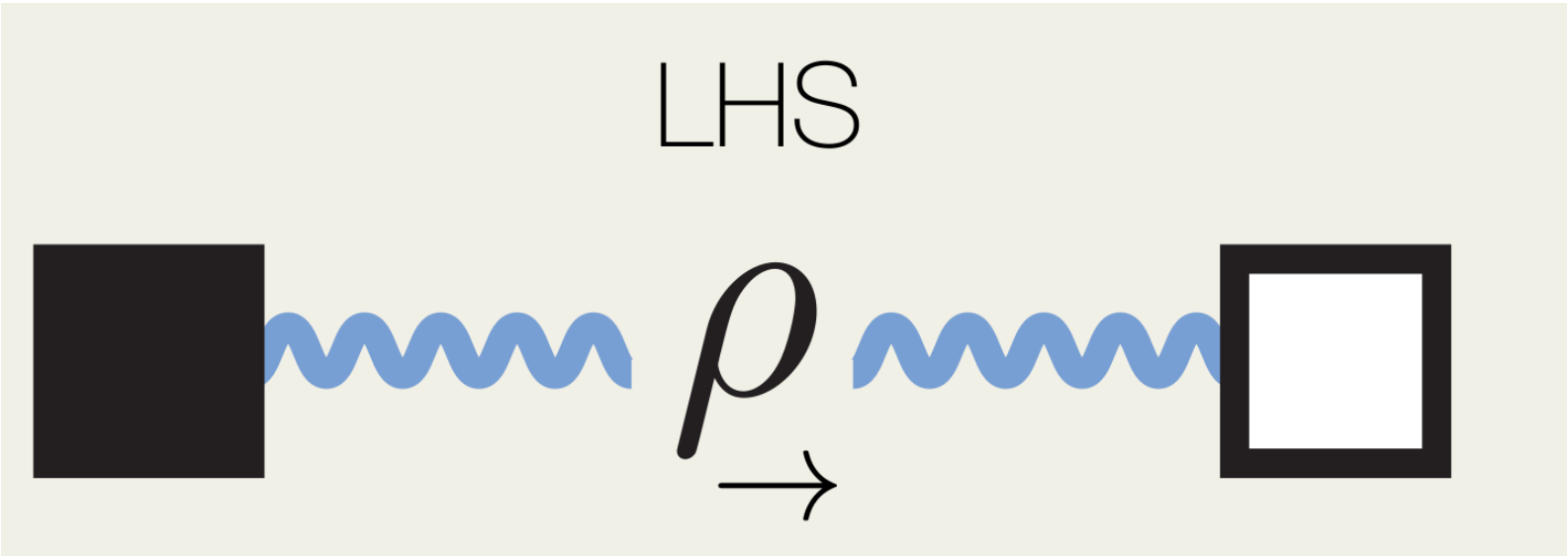



Inequivalence of entanglement, steering, and Bell nonlocality for general measurements
MT Quintino, T Vértesi, D Cavalcanti, R Augusiak, M Demianowicz, A Acín, N Brunner
Physical Review A, 92, 3, 2015

# Methods!

Step 2 - General results for local models

- Steering local models "respect" quantum mechanics in characterised part
- Sequential measurements and local operations are covered by the model

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# Methods!

Step 3 - Understading the Werner state after local filtering on Alice's side:

$$V\psi^- + (1-V)\frac{I}{4} \mapsto V\left|\psi_\theta\right\rangle\left\langle\psi_\theta\right| + (1-V)\psi_A \otimes \frac{I}{2},$$

$$\left|\psi_\theta\right\rangle := \cos\theta\left|00\right\rangle + \sin\theta\left|11\right\rangle, \quad \psi_A := \mathrm{tr}_B\,\psi_\theta$$

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Step 4 - Steering model:
We "just" need a steering model for

$$\rho = \left|\psi_\theta\right\rangle\!\left\langle\psi_\theta\right| + (1-V)\psi_A \otimes \frac{I}{2}$$

from Alice to Bob! ($\forall\theta$ and a fixed $V > 1/3$)

# Methods!

Projective measurements and $V \leq 1/2$

Sufficient criterion for guaranteeing that a two-qubit state is unsteerable

Joseph Bowles, Flavien Hirsch, Marco Túlio Quintino, and Nicolas Brunner
Phys. Rev. A **93**, 022121 – Published 26 February 2016

# Methods!

General POVMs and $V \leq 0.4$

**Algorithmic construction of local hidden variable models for entangled quantum states**

Flavien Hirsch, Marco Túlio Quintino, Tamás Vértesi, Matthew F. Pusey, Nicolas Brunner

*(Submitted on 1 Dec 2015)*

# Summarising

- Sequential measurements can reveal the nonlocality of local entangled states

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- Sequential measurements can reveal the nonlocality of local entangled states
- Sometimes two rounds with a single choice of measurement for the first round is enough
- Sometimes not
- Machinery developed for EPR-steering can be very useful for Bell nonlocality

# Future directions

- More choices of measurements on the first round

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- More choices of measurements on the first round
- Are all entangled states nonlocal in the sequential measurement framework?

# Thank you!