Joint measurability, EPR steering, and Bell nonlocality

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Phys. Rev. Lett. 113, 160402; Joint with: T. Vértesi, N. Brunner

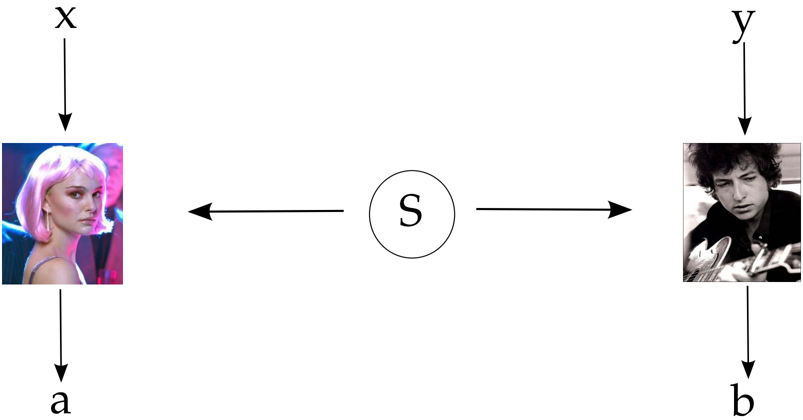
Joint Measurability

$$\Delta x \, \Delta p \geq \hbar/2$$

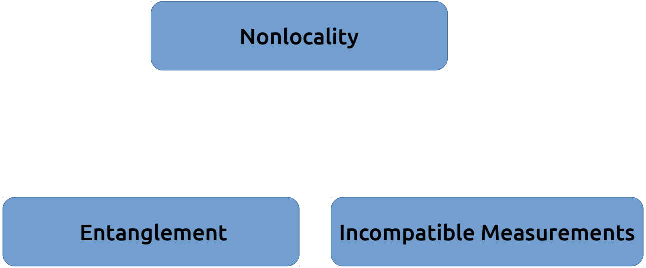
Entangled States

$$\rho_{AB} \neq \int \pi(\lambda) \rho_A^\lambda \otimes \rho_B^\lambda \, \mathrm{d}\lambda$$

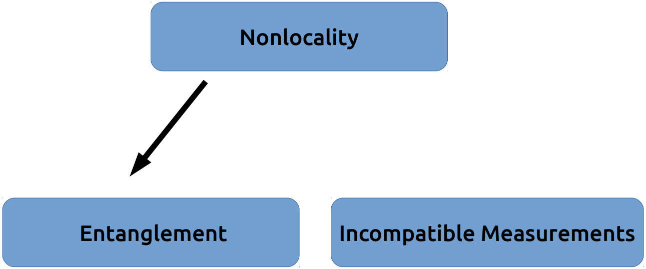
Nonlocality



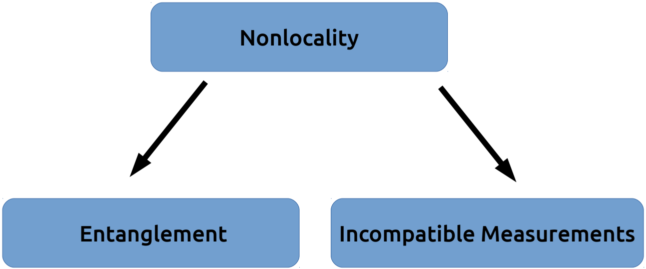
Relations between the concepts



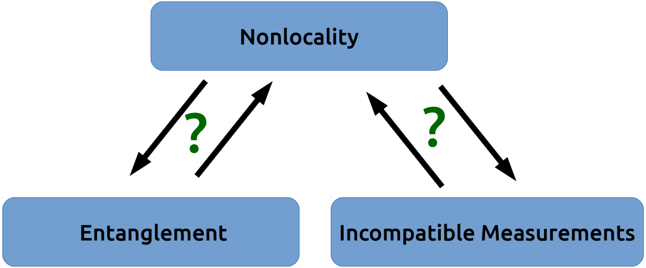
Separable states are local



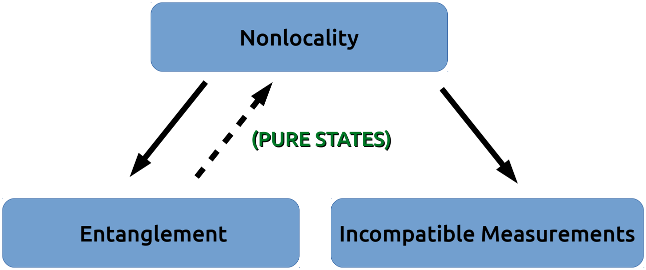
Compatible measurements lead to classical statistics



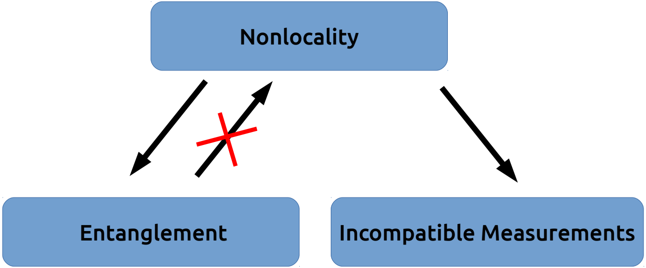
The converse question



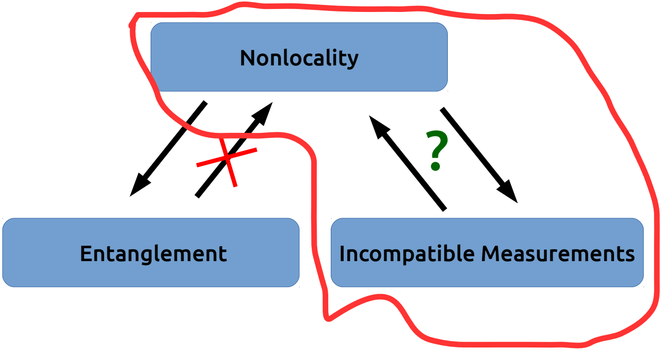
Pure states (N. Gisin, Phys. Lett. A 154, 201 (1991))



Werner States (1989)



Nonlocality and Quantum measurements



Compatible Measurements

- Quantum observables:

$$E = E^\dagger, \qquad F = F^\dagger$$

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- Jointly Measurability

More general measurements

- POVM:

$$E_e \geq 0, \quad \sum_a E_e = I$$

$$F_f \geq 0, \quad \sum_b F_f = I$$

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- POVM:

$$E_e \geq 0, \quad \sum_a E_e = I$$

$$F_f \geq 0, \quad \sum_b F_f = I$$

- Commutation of the POVM elements?

Joint Measurability

- $\{E_e\}$ and $\{F_f\}$ are JM if there exists a third measurement $\{G_{ef}\}$, such that

$$E_e = \sum_b G_{ef}\,, \quad F_f = \sum_a G_{ef}$$

Joint Measurability

- $\{E_e\}$ and $\{F_f\}$ are JM if there exists a third measurement $\{G_{ef}\}$, such that

$$E_e = \sum_b G_{ef}\,, \quad F_f = \sum_a G_{ef}$$

- By measuring $\{G_{ef}\}$ we get the output e and f

Pauli Measurements

$$\sigma_Z : \{|0\rangle\langle 0|\,,\ |1\rangle\langle 1|\} \quad \sigma_X : \{|+\rangle\langle +|\,,\ |-\rangle\langle -|\}$$

Noise Pauli Measurements

$$\sigma_{Z,\eta} : \left\{ \eta \left|0\right\rangle\left\langle 0\right| + (1-\eta)\frac{I}{2} \ ; \qquad \eta \left|1\right\rangle\left\langle 1\right| + (1-\eta)\frac{I}{2} \right\}$$

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$$\eta \leq \frac{1}{\sqrt{2}} \implies \text{Joint Measurability}$$

Hollow Triangle

$$\sigma_{Z,\eta} : \left\{ \eta \left|0\right\rangle\left\langle 0\right| + (1-\eta)\frac{I}{2} \; ; \qquad \eta \left|1\right\rangle\left\langle 1\right| + (1-\eta)\frac{I}{2} \right\}$$

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$$\sigma_{Y,\eta} : \left\{ \eta \left|Y+\right\rangle\left\langle Y+\right| + (1-\eta)\frac{I}{2} \; ; \qquad \eta \left|Y-\right\rangle\left\langle Y-\right| + (1-\eta)\frac{I}{2} \right\}$$

$$\eta \leq \frac{1}{\sqrt{2}} \implies \text{Pairwise Measurability}$$

Hollow Triangle

$$\sigma_{Z,\eta} : \left\{ \eta \left|0\right\rangle\left\langle 0\right| + (1-\eta)\frac{I}{2} \; ; \qquad \eta \left|1\right\rangle\left\langle 1\right| + (1-\eta)\frac{I}{2} \right\}$$

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$$\eta \leq \frac{1}{\sqrt{2}} \implies \text{Pairwise Measurability}$$

$$\eta \leq \frac{1}{\sqrt{3}} \implies \text{Triplewise Measurability}$$

Hollow triangle measurements

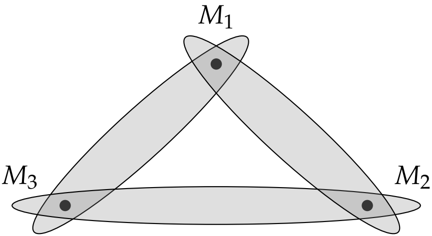
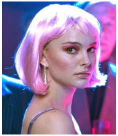


Figure: A “Hollow Triangle” measurement

EPR steering

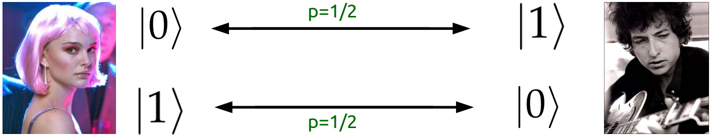
$$\psi^-$$

EPR steering

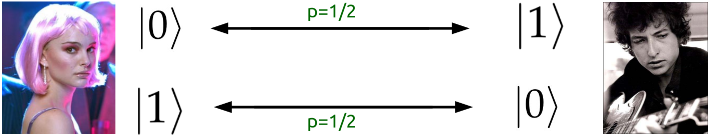


σ_Z ⟵ ψ^- ⟶

EPR steering

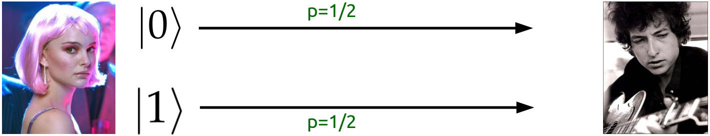


EPR steering

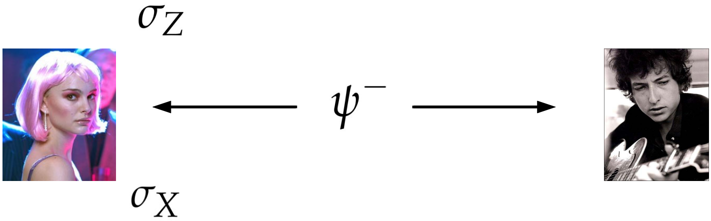


Quantum steering?

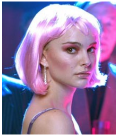
How to fool Bob



EPR steering



EPR steering



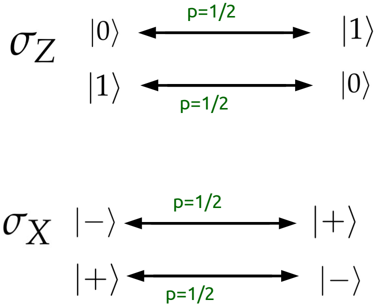
σ_Z $|0\rangle$ ⟷ $|1\rangle$ (p=1/2)

$|1\rangle$ ⟷ $|0\rangle$ (p=1/2)

σ_X $|-\rangle$ ⟷ $|+\rangle$ (p=1/2)

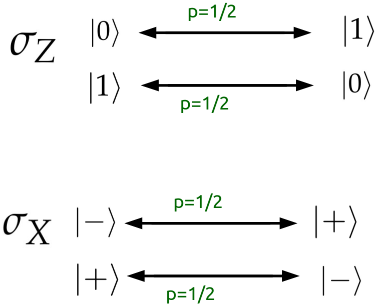
$|+\rangle$ ⟷ $|-\rangle$ (p=1/2)

EPR steering



This *cannot* be simulated by classical mixtures of qubits.

EPR steering



Incompatible measurements + Entangled state –> EPR steering

Assemblage

Bob's state after $A_{a|x}$

$$\rho_{a|x} = \frac{\mathrm{tr}_A(\rho_{AB} A_{a|x} \otimes I)}{p(a|x)}$$

Assemblage

Bob's system is completely described by an *assemblage*:

$$\sigma_{a|x} = \mathrm{tr}_A(\rho_{AB} A_{a|x} \otimes I)$$

$$\rho_{a|x} = \frac{\sigma_{a|x}}{\mathrm{tr}(\sigma_{a|x})}$$

Unsteerable Assemblages

Classical mixture of single part quantum states:

$$\rho_{a|x} = \sum_{\lambda} \pi(\lambda|a, x)\rho_{\lambda}$$

Unsteerable Assemblages

When an assemblage is unsteerable?

$$\rho_{a|x} = \sum_{\lambda} \pi(\lambda|a, x)\rho_{\lambda}$$

Bayes Rule:

$$\pi(\lambda|a, x) = \frac{p_A(a|x, \lambda)\pi(\lambda)}{p(a|x)}$$

Unsteerable Assemblages

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Bayes Rule:

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And we have:

$$\rho_{a|x} = \sum_{\lambda} \frac{p_A(a|x,\lambda)\pi(\lambda)}{p(a|x)}\rho_{\lambda}$$

Unsteerable Assemblages

$$\sigma_{a|x} = \sum_{\lambda} \pi(\lambda) p_A(a|x, \lambda) \rho_\lambda$$

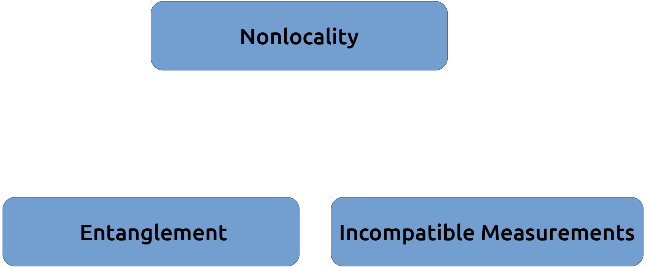
Separable states are EPR local

- Only entangled states can lead to steerable assemblages

Separable states are EPR local

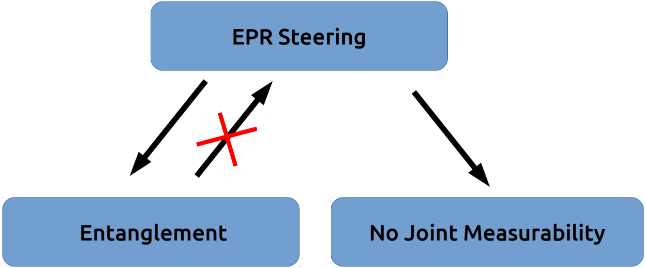
- Only entangled states can lead to steerable assemblages
- Entanglement certification (Even when Bob does not trust Alice)

Relations between the concepts

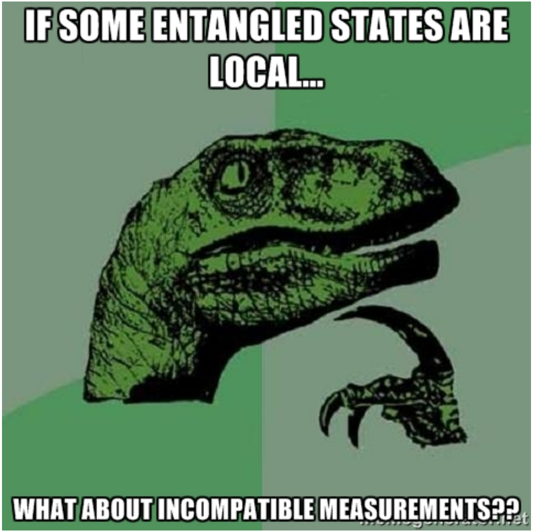


Werner states (1989)

Some entangled states are EPR local!



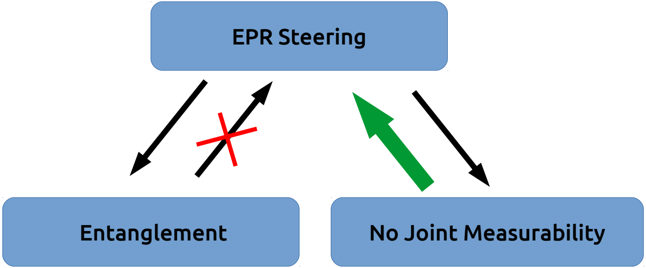
Local Incompatible Measurements??



All incompatible measurements can lead to steering!

$$\text{No Joint Measurability} \iff \text{Useful for EPR Steering}$$

Our contribution



The precise result

Theorem

Let $\{A_{a|x}\}$ be a set of incompatible measurements. There exists a quantum state ρ_{AB} such that the assemblage $\sigma_{a|x} = \mathrm{tr}_A(A_{a|x} \otimes I \rho_{AB})$ is steerable.

The precise result

Theorem

Let $\{A_{a|x}\}$ be a set of incompatible measurements. If Alice measures $\{A_{a|x}\}$ on her part of a pure entangled state, the resulting assemblage is steerable.

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Theorem

Let $\{A_{a|x}\}$ *be a set of incompatible measurements. If Alice measures* $\{A_{a|x}\}$ *on her part of a pure entangled state, the resulting assemblage is steerable.*

Any number of measurements, any number of outputs, any quantum dimension.

New interpretations for Quantum Joint Measurability

We can now interpret joint measurability in terms of EPR correlations.

Applications

- Explore known results from the Steering community to get results for Joint Measurability

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- More (tight) unsteerable states:

$$\rho_{UNS} = \frac{1}{2}\Phi_\theta + \frac{1}{2}\frac{I}{2} \otimes \rho_{B,\theta}$$

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- More (tight) unsteerable states:

$$\rho_{UNS} = \frac{1}{2}\Phi_\theta + \frac{1}{2}\frac{I}{2} \otimes \rho_{B,\theta}$$

- $\eta \leq 5/12 \implies$ all POVMs can be measured (Barrett 2002 + Quintino *et al* 2015 + this work -> Pusey 2015)

Applications

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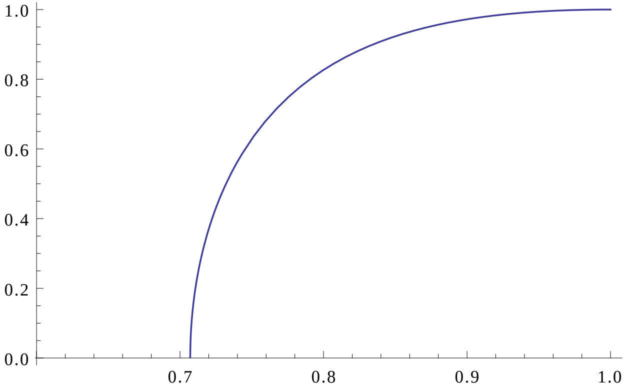
- What is the probability of Bob having an steering assemblage when Alice perform random measurements?

Applications

- Alice and Bob share a two qubit Werner state
- What is the probability of Bob having an steering assemblage when Alice perform random measurements?
- If she performs two uniformly random projective ones:

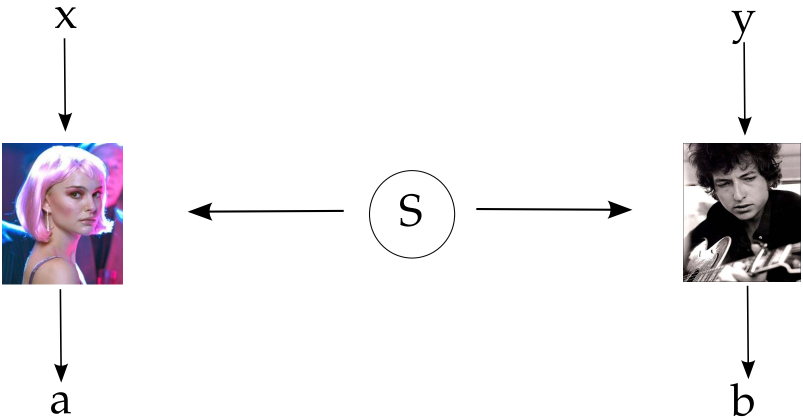
Applications

$$p(\eta) = \sqrt{\frac{2\eta^2 - 1}{\eta^4}}$$

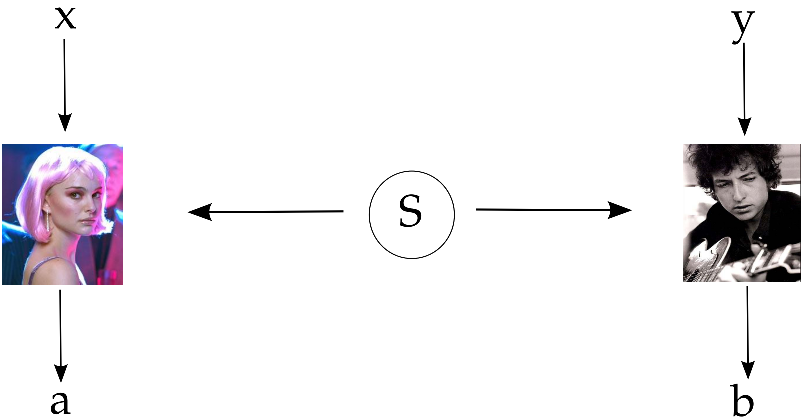


Busch, P. (1986) Unsharp reality and joint measurements for spin observables. Phys. Rev. D 33)

Bell Nonlocality



Bell Nonlocality



$$p(ab|xy)$$

Bell Inequalities

Bell Nonlocality can be witnessed by Bell Inequalities

$$CHSH = \langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle \leq 2$$

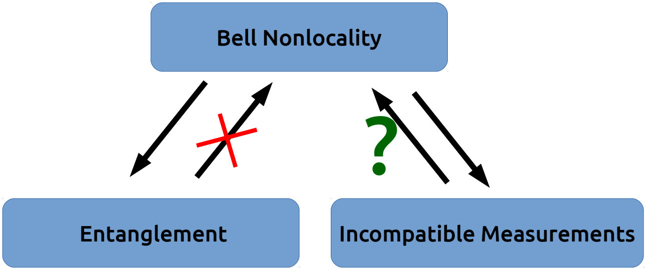
Bell Inequalities

Bell Nonlocality can be witnessed by Bell Inequalities

$$CHSH = \langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle \leq 2$$

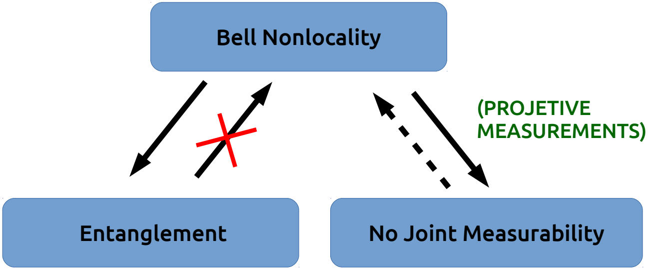
$$\langle A_x B_y \rangle := p(a = b|xy) - p(a \neq b|xy)$$

Incompatible measurements and Bell Nonlocality



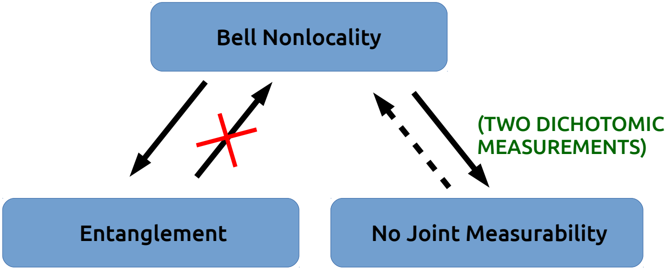
Projective Measurements

L.A. Khalfin, B.S. Tsirelson, 1985



Two dichotomic measurements

M. M. Wolf, D. Perez-Garcia, C. Fernandez, 2009



Incompatible measurements and Bell Nonlocality

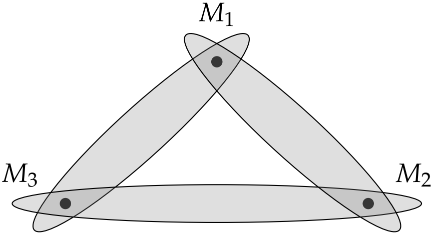
- There may exist incompatible Bell local measurements . . .

Incompatible measurements and Bell Nonlocality

- There may exist incompatible Bell local measurements ...
- Evidence 1: Maximally entangled state $\implies$ incompatible Bell local measurements

Incompatible measurements and Bell Nonlocality

- There may exist incompatible Bell local measurements ...
- Evidence 1: Maximally entangled state $\implies$ incompatible Bell local measurements
- Evidence 2: Hollow triangle measurements + full-correlation type cannot show bell nonlocality
  $(\langle A_x B_y \rangle := p(a = b|xy) - p(a \neq b|xy))$



Incompatible measurements and Bell Nonlocality

Conjecture

There exists a set of non Jointly Measurable measurements that can never lead to Bell inequality violation.

Conclusions

- Better understanding on the relation between quantum measurements and nonlocality

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- Better understanding on the relation between quantum measurements and nonlocality
- Conceptually: How to interpret JM in terms of EPR steering (vice versa!)
- Applications: Some theorems for JM can be translated to Nonlocality (vice versa!)

Open questions

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- Can all incompatible measurements lead to Bell nonlocality?

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- Which nontrival results can we get by exploring this connection?
- Can all incompatible measurements lead to Bell nonlocality?
- If our conjecture is correct, is there a simple concept that captures Bell Nonlocality for quantum measurements?

After finishing

"Joint Measurability of generalized measurements implies classicality"
R. Uola, T. Moroder, O. Gühne
Phys. Rev. Lett. 113, 160403

Thank you!